New Trends in Mechanism Design for Considering Participants' Interactions

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Outline

1. Influential Agent Selection in Networks
   - Deterministic Mechanisms
   - Random Mechanisms
2. Cost Allocation

How to select the most influential k agents?

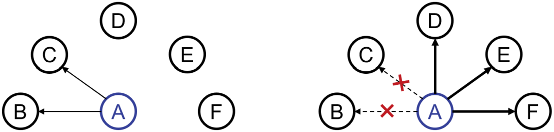
- Influential agent can be defined by progeny, centrality (e.g., (in)-degree, betweenness, etc.), PageRank, etc..
- Known: all agents in the network;
  Unknown: true relationships among agents.

Question

How to select the most influential k agents with their reported relationships, s.t. agents will NOT misreport their relationships?

How to select the most influential k agents?

- Agents' manipulation space: add/delete edges arbitrarily.



Challenge

Agents may misreport to make themselves selected, which means that we cannot simply trust their reports, so the selection mechanism cannot be optimal in order to avoid misreporting.

Two Kinds of Selection Mechanisms

Deterministic Mechanisms

Map a given graph to a k-subset of selected agents.

Random Mechanisms

Map a given graph to a probability distribution on all possible k-subsets of agents.

Outline

From (In)-degree Perspective

Sum of Us: Strategyproof Selection from the Selectors [Alon et al., TARK'11]

- Randomly select k agents with as high degrees as possible.
- Deterministic k-selection mechanisms by degree is IC and has a non-zero approximation ratios iff all n agents are selected, i.e. $k = n$.

Definition

Denote the k agents selected by a mechanism $\mathcal{M}$ in a graph G as $i_1, ..., i_k$ and $deg(i) = deg(i, G)$ as the in-degree of agent i in G. Let the optimal selection be $j_1, ..., j_k$, then the approximation ratio is $\frac{\mathbb{E}(deg(i_1, ..., i_k))}{deg(j_1, ..., j_k)}$.

From (In)-degree Perspective

Suppose $n = 2, k = 1$

- Select the first k agents
  A has an incentive to misreport:

Not IC!

- Select fixed k agents
  always select B no matter what A's strategies are:

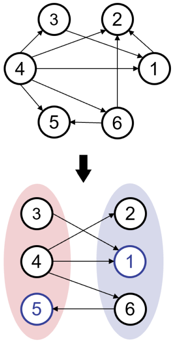
IC but no guarantee of non-zero bounds!

Outline

The Random *m*-Partition Mechanism (*m*-RP)

Main Idea

Divide all agents into subsets, and agents can only be selected by the connections from the other subsets.



- Randomly partition the set of agents into m subsets.
- Roughly select the top $\frac{k}{m}$ agents in each subset with the largest in-degrees from the other subsets.
- Given $k = 2$, $m = 2$, the left graph is divided into two subsets $\{3, 4, 5\}$ and $\{2, 1, 6\}$. The selected is $\{1, 5\}$, while the optimal is $\{2, 5\}$.

The Random m-Partition Mechanism (m-RP)

For a given k, we can choose the best value of m by m-RP:

- 2-RP provides an approximation ratio of $\frac{1}{4}$.
- For $k \geq 2$, $\lceil k^{1/3} \rceil$-RP provides an approximation ratio of $\frac{1}{1+\mathcal{O}(1/k^{1/3})}$.

Question

Can we improve the approximation ratio?

Bound Optimization by Degree

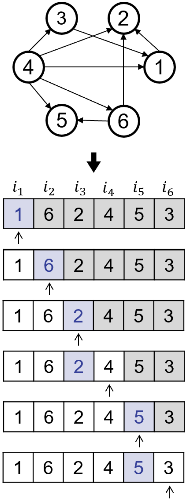
Optimal Impartial Selection [Fischer and Klimm, SIAM J. Comput'15]

- Extend the random partition method.
- $\frac{1}{2}$-optimal approximation ratio when selects a single agent by degree.

Main Idea

Use the 2-partition method many times to gradually improve the selection.

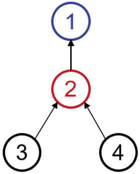
The Permutation Mechanism



- Put n agents in a random order $i_1, \ldots, i_n$.
- Traverse all agents and compares the current agent i_m with $i_1, \ldots, i_k, \ldots, i_{m-1}$.
  - the indegree of the selected i_k: $d(i_k)$ = outdegree of $i_1, \ldots, i_{k-1}$ to i_k.
  - the indegree of i_m: $d(i_m)$ = outdegree of $i_1, \ldots, i_{m-1}$ to i_m, except i_k.
- If $d(i_m) \geq d(i_k)$, then i_m becomes the newly selected agent.
- For the left graph, with random permutation $\{1, 6, 2, 4, 5, 3\}$ and the selected agent is 5.
- The mechanism achieves $\frac{1}{2}$-optimal.

In-degree is not always the right measure

Consider to find an influential user on Twitter for a product promotion.



- measure influence by degree: the user with the highest degree is 2.
- the most influential user here maybe 1.

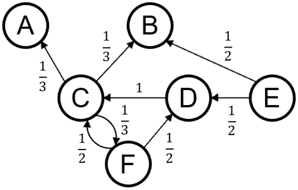
Solution

Take both direct and propagated influence into account.

From Diffusion Perspective

Incentive-Compatible Diffusion [Babichenko et al., WWW'18]

- Select the most influential agent in the network.
- Agent i's influence $I(i)$: the probability of reaching her in a random walk.



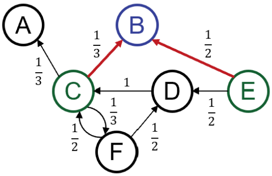
- $I(x, y)$: the influence of x on y, i.e. the probability of reaching x from y.
- The influence of C:

$$I(C, A) = I(C, B) = 0;$$
$$I(C, C) = I(C, D) = 1;$$
$$I(C, E) = 1 \cdot \frac{1}{2} = \frac{1}{2};$$
$$I(C, F) = \frac{1}{2} + 1 \cdot \frac{1}{2} = 1;$$
$$\Rightarrow I(C) = 0 + 1 + 1 + \frac{1}{2} + 1 = 3\frac{1}{2}$$

The Two Path Mechanism

Main Idea

Inspired by *the friendship paradox*, i.e. neighbors always have better diffusion influence in expectation; an agent is more likely to be selected if more random walks can reach her.



- Start two independent random paths from two randomly chosen agents.
- If the two paths do not intersect, then delete all agents in these paths; If they intersect at an agent, then the agent is selected.
- E.g. start from C, E, C reaches B with a probability of $\frac{1}{3}$ and E with $\frac{1}{2}$. So B can be selected with a probability of $\frac{1}{6}$.

The Two Path Mechanism

Properties:

- $\frac{2}{3}$-approximation ratio on a tree.
- No guarantee in forests and directed acyclic graphs (DAGs).

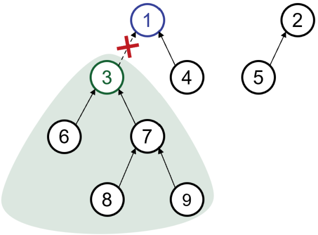
Question

Is there another way to take both direct and propagated influence into account?

From Progeny Perspective

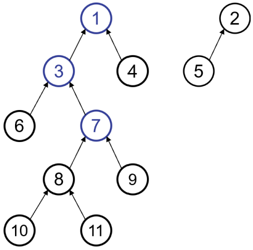
Incentive-Compatible Selection Mechanisms for Forests [Babichenko et al., EC'20]

- Select the most influential agent in a forest.
- Define agent *i*'s influence by the number of her progeny $\mathcal{P}_i$.



- 1 is the most influential agent with the max $\mathcal{P}_i$ in the forest.
- 3 becomes most influential by deleting her out-edge.

From Progeny Perspective: The Mechanism

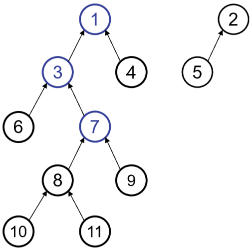


- Influential Node i: agent with max $\mathcal{P}_i$ after deleting her out-edges.
- Influential Set $\mathcal{A}$: all influential nodes in the network, e.g. $\{1, 3, 7\}$.
- Sub-influential Set $\mathcal{A}_i$: the influential set without i's out-edges, e.g. $\mathcal{A}_3 = \{3, 7\}$.

Main Idea

Give a selection probability distribution $\mathbf{x}$ to all the influential nodes in $\mathcal{A}$.

From Progeny Perspective: The Mechanism



- Find the influential set $\mathcal{A}$, $\{1, 3, 7\}$.
- Define the probability distribution as:

$$x_i = \begin{cases} 1/2, & |\mathcal{A}_i| = 1 \\ 1/2log_2\left(\mathcal{P}_i/\overline{\mathcal{P}_i}\right), & |\mathcal{A}_i| > 1 \end{cases}$$

  where $\overline{\mathcal{P}_i}$ is the max progeny of i's children.
- we have $x_1 = \frac{1}{2}log_2\frac{9}{7}$, $x_3 = \frac{1}{2}log_2\frac{7}{5}$, $x_7 = \frac{1}{2}$, $x_i = 0$ for the others.

From Progeny Perspective: The Mechanism

Properties:

- About $\frac{1}{3}$-approximation ratio in forests, not for DAGs.
- A proved upper bound of $\frac{4}{5}$-approximation ratio for any IC selection mechanism by the number of progeny in all networks.

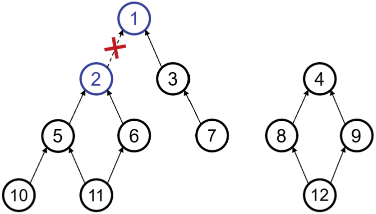
Question

Whether we can achieve a good approximation ratio in DAGs?

Selection in DAGs

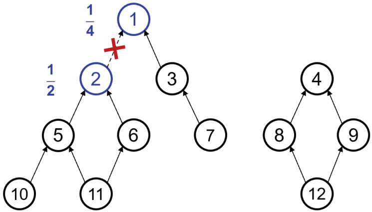
Incentive Compatible Mechanism for Influential Agent Selection [Zhang et al. SAGT'21]

- A DAG forms when agents join sequentially, e.g. a reference network.



- Influential Set: $\{1, 2\}$
- Restrict agents' manipulation space: an agent can only report a subset of her real out-edges.

Selection in DAGs: The Geometric Mechanism



- Give agents' reports and find the influential set $\{1, 2\}$
- Assign the selection probability on the influential set $\{k, k-1, ..., 1\}$ as:

p_k	p_{k-1}	...	p_1
$\frac{1}{2}$	$\frac{1}{4}$	...	$\frac{1}{2^k}$

- so $p_1 = \frac{1}{4}$, $p_2 = \frac{1}{2}$, $p_i = 0$ for the others.

Selection in DAGs: The Properties

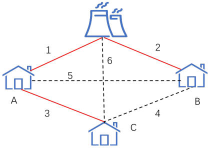
- Geometric Mechanism
  - $\frac{1}{2}$- approximation ratio.
  - Incentive compatibility.
  - Fairness.
- A proved upper bound of $\frac{1}{1+\ln 2}$-approximation ratio for any incentive compatible and fair selection mechanism.
- Geometric Mechanism achieves an approximation ratio close to the upper bound.

Future Work

- Narrow the gap between the ratio of the Geometric Mechanism and the proved upper bound.
- Extend to settings where more than one agent need to be selected.

Outline

The Cost Allocation Problem



- A set of agents located at different locations and there is a source.
- The agents need to connect to the source directly or indirectly.
- The cost of connecting any pair of agents is known and the total cost has to be shared among all connected agents.
- Q: How to allocate the cost among all connected agents?

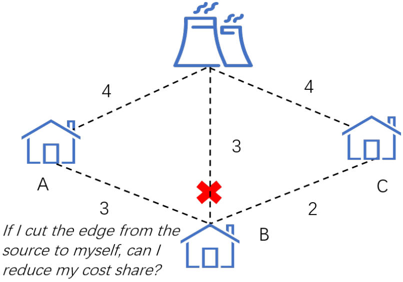
Ref: Tijs and Driessen 1986: *Game theory and cost allocation problems*.

Agents' Strategic Behaviour

- For connecting to the source, an agent may need to go through other agents, the intermediate agents can strategically cut the connection by cutting the edges adjacent to them.
- This is a very practical issue if cutting edges can reduce their cost share.
- Such strategic behaviour will potentially increase the total cost of connecting all agents.

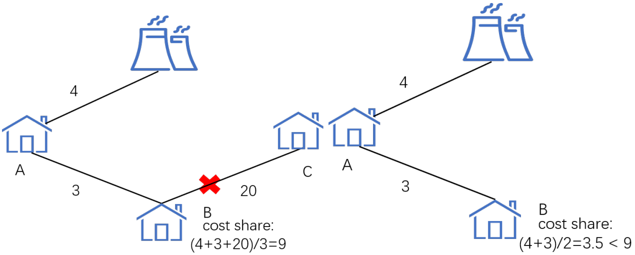
The Goal

We need to design cost allocation rules to prevent such strategic behaviour.



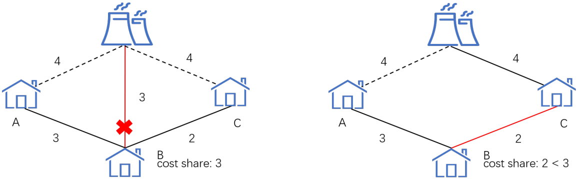
Simple Ideas

- The total cost is shared equally.
    - Cannot prevent nodes from cutting their adjacent edges.



Simple Ideas

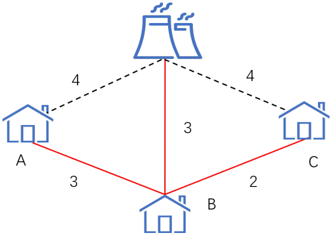
- Each node pays the cost of its connected edge corresponding to the minimum spanning tree (MST).
  - Cannot prevent nodes from cutting their adjacent edges.



A Good Solution

Marginal cost to join the game:

- The nodes with larger distance/depth to the source join the game earlier
    - In the following example, consider the join order of A, B, C. The marginal cost for them are $4, 2, 2$.

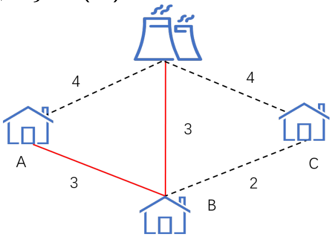


Average on All Permutations? (Shapley Value)

Definition (Characteristic Function)

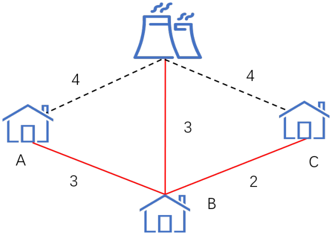
For each subset S, $v(S) \in \mathcal{R}^+$ is the minimal cost of connecting S. We can use the nodes outside S to compute $v(S)$.

For the set $S = \{A, B\}$, $v(S) = 3 + 3 = 6$.



Shapley Value

- The values for the example are: $v(A) = 4$, $v(B) = 3$, $v(C) = 4$, $v(A, B) = 6$, $v(A, C) = 8$, $v(B, C) = 5$, $v(A, B, C) = 8$, $v(\emptyset) = 0$.
- The average marginal cost for A, B, C are $3.5, 1.5, 3$.



Properties

- Truthful (prevent nodes from cutting their adjacent edges).
- Budget Balanced (the total cost share equals the total cost).
- Cost Monotonic (each node's cost share weakly increases if the cost of one of its adjacent edges increases.)
- Positive (each node's cost share is non-negative.)

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