New Trends in Mechanism Design for Considering Participants' Interactions

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A tutorial @ AAAI 2022

2009 DARPA Red Balloon Challenge

- The $40,000 challenge award would be granted to the first team to submit the locations of 10 moored, 8-foot, red weather balloons at 10 previously undisclosed fixed locations in the continental United States.

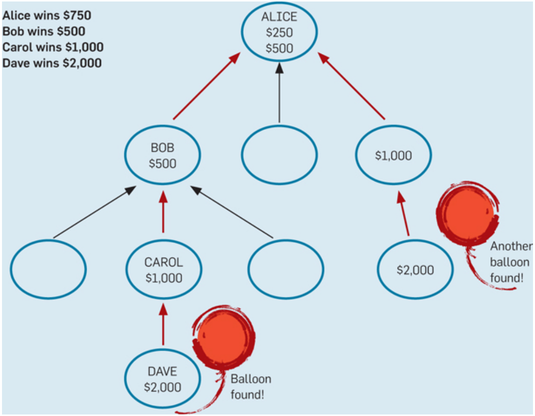
2009 DARPA Red Balloon Challenge

MIT Crowdsourced Solution (The Winner):

- "*We're giving $2000 per balloon to the first person to send us the correct coordinates, but that's not all – we're also giving $1000 to the person who invited them. Then we're giving $500 whoever invited the inviter, and $250 to whoever invited them, and so on ...*"
- got over 5,000 of participants, won the competition in under 9 hours.

2009 DARPA Red Balloon Challenge

MIT Crowdsourced Solution (The Winner):



- Pickard, G., et al., Time-Critical Social Mobilization. Science, 2011. 334(6055): p. 509-12.

PinDuoDuo (like Groupon)

What are the incentives?

More participants, higher chance to win!!!

- 2009 DARPA Red Balloon Challenge
  - Inviting more friends has higher chance to win (higher utility)
- PinDuoDuo
  - Inviting more friends has higher chance to get cheap items (higher utility)

What if it is a competition?

- Resource allocation (auctions)
- Task allocation (crowdsourcing)
- Matching
- Resource sharing

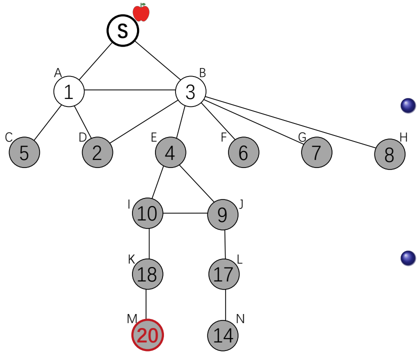
More participants means lower chance to win!!!

Diffusion Mechanism Design

Mechanism Design on Social Networks

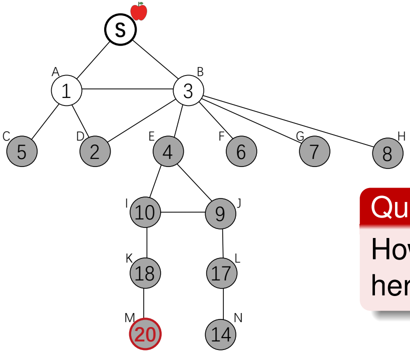
Design mechanisms/markets under competitive environment such that participants are incentivized to invite more participants/competitors to join the mechanisms.

Starter: Promote a Sale via Social Networks



- The seller sells one item and has only two connections/neighbours in the network (A,B).
- Each node is a potential buyer and the value is her highest willing payment to buy the item (valuation).
- The seller's revenue of applying second price auction (VCG) without promotion is 1.
- but the highest willing payment in the network is 20.

Starter: Promote a Sale via Social Networks



Question

How the seller could do to increase her profit?

Traditional Sale Promotions

Traditional sale promotions:

- Promotions via agents
- Keywords based ads via search engines such as Google
- Ads via social media such as WeChat, Facebook, Twitter

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Challenge

- The return of these promotions are unpredictable.
- The seller may LOSE from the promotions.

Tackle the Challenge

Build promotion inside the market mechanism such that

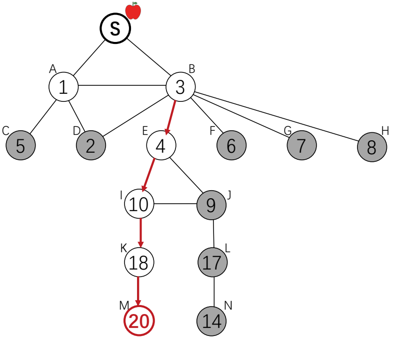
1. the promotion will never bring negative utility/revenue to the seller.
2. all buyers who are aware of the sale are incentivized to diffuse the sale information to all her neighbours.

"Diffusion Mechanism Design"

New Challenges

Why a buyer would bring more buyers to compete with her?

- only if their diffusion are rewarded, *but the seller doesn't want to lose*!
- we cannot just pay each node a fixed amount to incentivise them to diffuse the information.



Outline

Outline

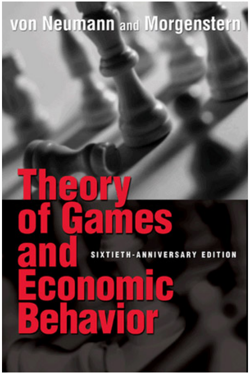
What is Mechanism Design

What is Mechanism/Market Design?

- it is known as Reverse Game Theory

What is Game Theory

- **Game theory** is the study of mathematical models of conflict and cooperation between intelligent rational decision-makers (wiki) [von Neumann and Morgenstern 1944].



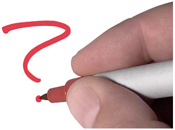
- Non-cooperative games: Go, poker, rock-paper-scissors
- Cooperative games: coordination games

Mechanism Design (Reverse Game Theory)

Mechanism Design is to answer...

Question

How to design a mechanism/game, toward desired objectives, in strategic settings?

	Participants		Game		Outcome
Game Theory	✓	⇒		⇒	
Mechanism Design	✓	⇒		⇒	✓

Mechanism Design (Reverse Game Theory)

Mechanism Design is to answer...

Question

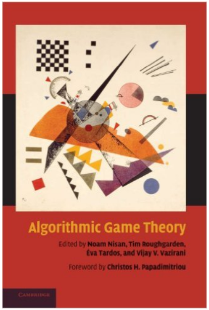
How to design a mechanism/game, toward desired objectives, in strategic settings?

Roger B. Myerson (born March 29, 1951, University of Chicago, US)

- Nobel Prize for economics (2007), for "having laid the foundations of mechanism design theory."
- *Eleven game-theorists have won the economics Nobel Prize*.

Algorithmic Game Theory (AGT)

- **Algorithmic game theory** is an area in the intersection of game theory and algorithm design, whose objective is to design algorithms in strategic environments (wiki) [Nisan et al. 2007].



- *Computing in Games*: algorithms for computing equilibria
- *Algorithmic Mechanism Design*: design games that have both good game-theoretical and algorithmic properties
- ...

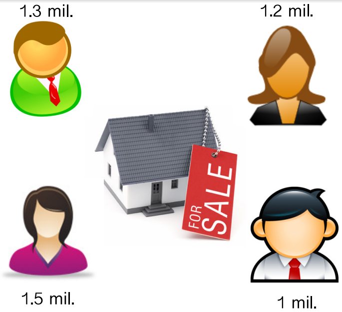
Algorithmic Game Theory in Artificial Intelligence

- Algorithmic game theory research in AI:
  - Game Playing: computation challenges, AlphaGo, poker
  - Social Choice: preferences aggregation, voting, prediction
  - Mechanism Design: the allocation of scarce resources, ad auctions
- Many IJCAI Computers and Thought Award (outstanding young scientists in artificial intelligence) winners had worked on AGT:
  - Sarit Kraus (1995), Nicholas Jennings (1999), Tuomas Sandholm (2003), Peter Stone (2007), Vincent Conitzer (2011), Ariel Procaccia (2015), and Fei Fang (2021)

A Mechanism Design Example

Design Goal

How can a house-seller sell her house with the "highest" revenue?



- **Challenge**: the seller doesn't know how much the buyers are willing to pay (their valuations).

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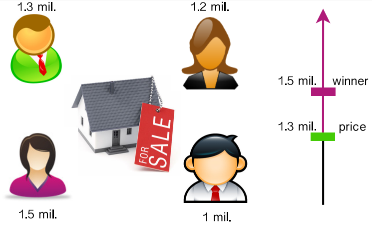

Solution: Second Price Auction (Vickrey Auction/VCG)

- Input: each buyer reports a price/bid to the seller
- Output: the seller decides
  - *allocation*: the agent with the highest price wins.
  - *payment*: the winner pays the second highest price.

A Mechanism Design Example

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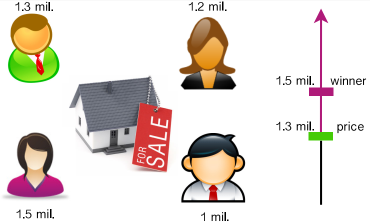

Solution: Second Price Auction (Vickrey Auction/VCG)

Properties:

- Efficient: maximising social welfare
- Truthful: buyers report their valuations truthfully

Is this the BEST the seller can do?

Question

What can the seller do to FURTHER increase her profit?

- estimate a good reserve price [Myerson 1981]
  - *requires a good estimation of buyers' valuations*
- promotions: let more people know/participate in the auction

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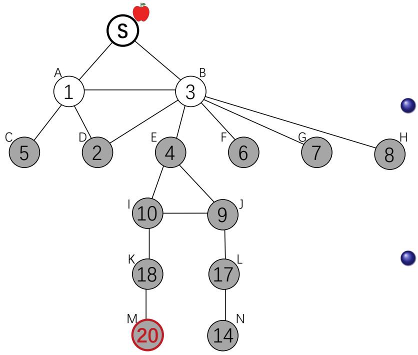
Recap: Promote a Sale via Social Networks



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Outline

How to Incentivize Participants to Invite More Participants in Resource Allocation?

Participants can get a higher utility by inviting more participants.

- “resell" the resource to their invitees
- have a higher chance of winning the resource by inviting more participants

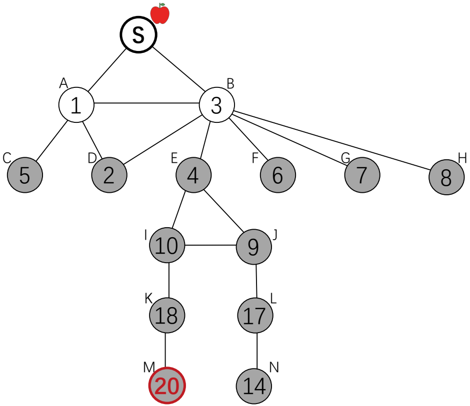
New Solutions: Information Diffusion Mechanisms

- Bin Li, Dong Hao, Dengji Zhao, Tao Zhou: Mechanism Design in Social Networks. AAAI'17.
- Dengji Zhao, Bin Li, Junping Xu, Dong Hao, Nick Jennings: Selling Multiple Items via Social Networks. AAMAS'18.
- Bin Li, Dong Hao, Dengji Zhao, Makoto Yokoo: *Diffusion and Auction on Graphs*. IJCAI'19.
- Wen Zhang, Dengji Zhao, Hanyu Chen: *Redistribution Mechanism on Networks*. AAMAS'20.
- Wen Zhang, Dengji Zhao, Yao Zhang: *Incentivize Diffusion with Fair Rewards*. ECAI'20.
- Bin Li, Dong Hao, Dengji Zhao: Incentive-Compatible Diffusion Auctions. IJCAI'20.
- Takehiro Kawasaki et al.: *Strategy-Proof and Non-Wasteful Multi-Unit Auction via Social Network*. AAAI'20.
- Bin Li, Dong Hao, Hui Gao, Dengji Zhao: Diffusion Auction Design. Artif. Intell. 303: 103631 (2022)

Outline

The First Diffusion Auction

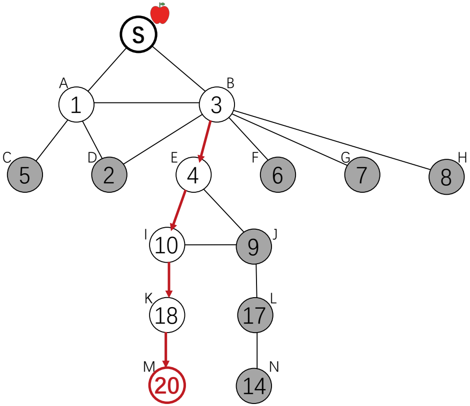
- Bin Li, Dong Hao, Dengji Zhao, Tao Zhou: **Mechanism Design in Social Networks**. AAAI'17.



Information Diffusion Paths

An information diffusion path from the seller to node M:

$s \rightarrow B \rightarrow E \rightarrow I \rightarrow K \rightarrow M$



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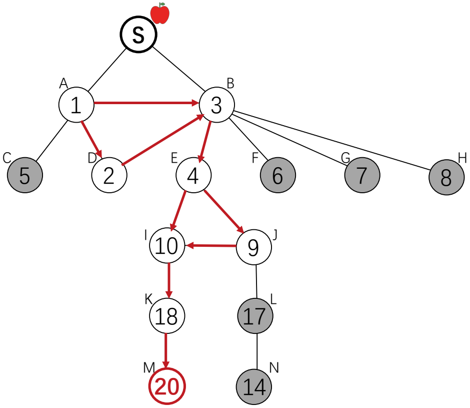
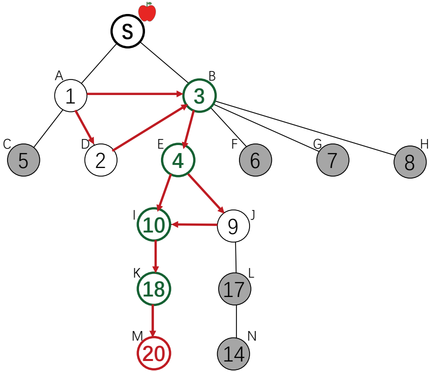


Diffusion Critical Nodes



Definition

i is *j*'s diffusion critical node if all the information diffusion paths started from the seller *s* to *j* have to pass *i*.

- nodes **B**, **E**, **I** and **K** are **M**'s only diffusion critical nodes.

Information Diffusion Mechanism [Li et al. AAAI'17]

The payment definition (second-price-like):

- If a buyer or one of her "*diffusion critical children*" gets the item, then the buyer pays the highest bid of the others (without the buyer's participation);
- otherwise, her payment is zero.

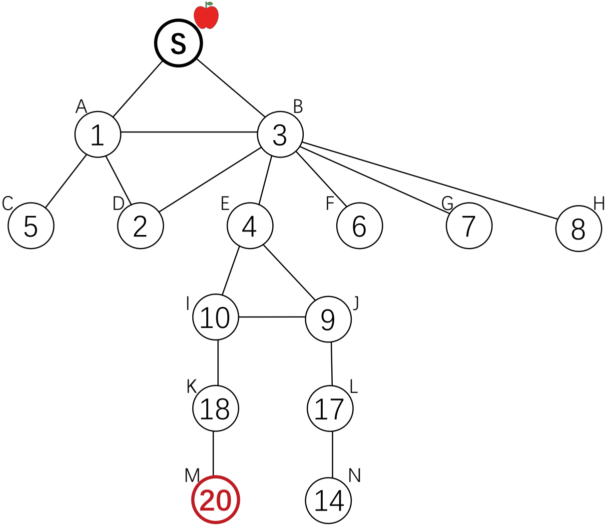
Information Diffusion Mechanism

The allocation definition:

- Identify the node i with the highest bid and the node's diffusion critical node path $P_{c_i} = (c_i^1, c_i^2, ..., i)$.
- Give the item to the first node of P_{c_i}, the node pays to the seller and then decides to whether keep the item or pass it to the next node in P_{c_i}:
  - If the payment of the next node is greater than the bid of the current node, passes it to the next node and receives the payment from the next node; the next node makes a similar decision;
  - otherwise, keep the item.

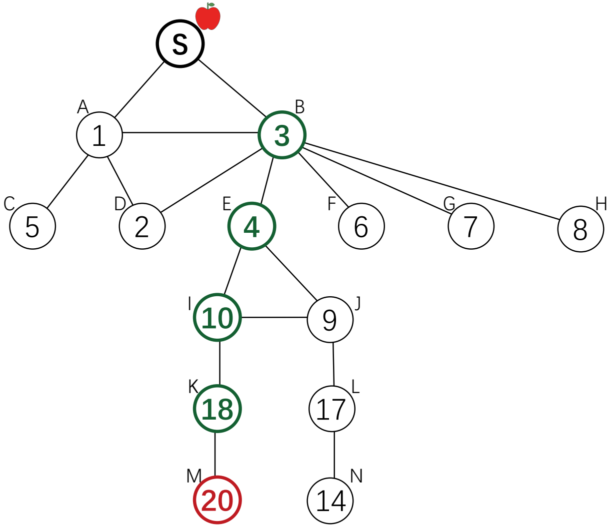
Information Diffusion Mechanism

find the highest bidder M



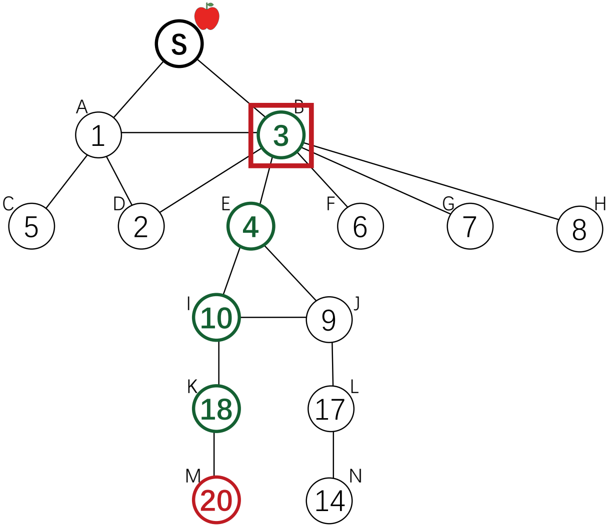
Information Diffusion Mechanism

find the diffusion critical path (B,E,I,K) to M



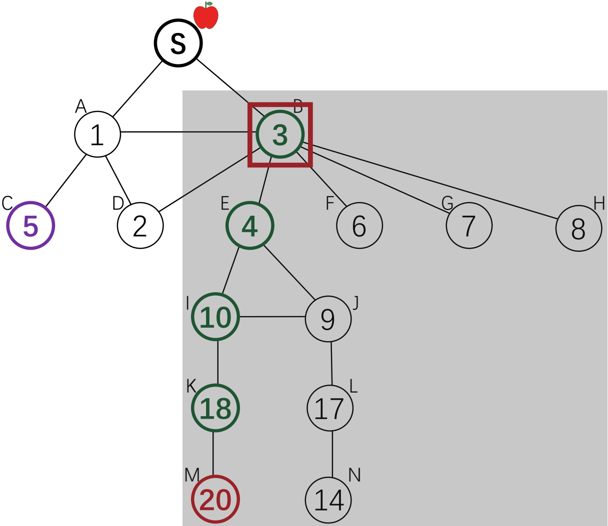
Information Diffusion Mechanism

pass the item to B



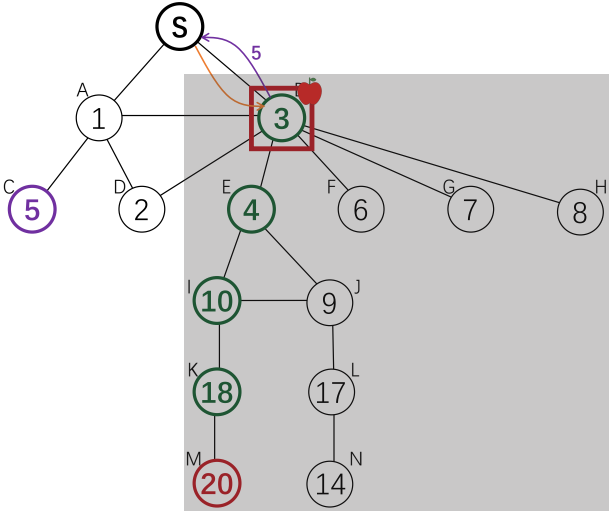
Information Diffusion Mechanism

compute the payment of B by removing B



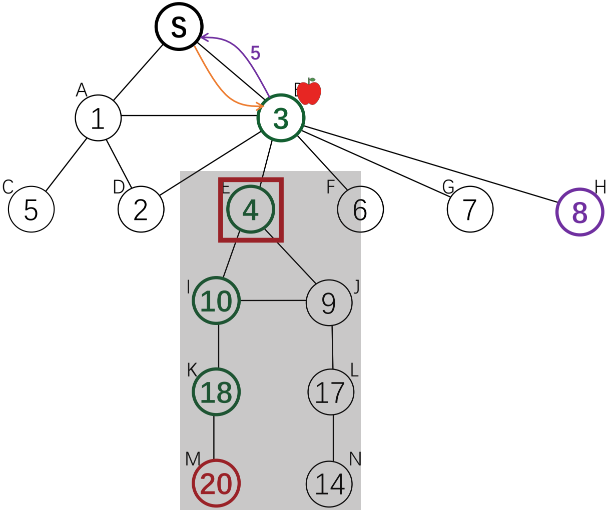
Information Diffusion Mechanism

the highest bid of the remaining bidders is 5



Information Diffusion Mechanism

so B pays 5 to get the item and decides to resell it or not



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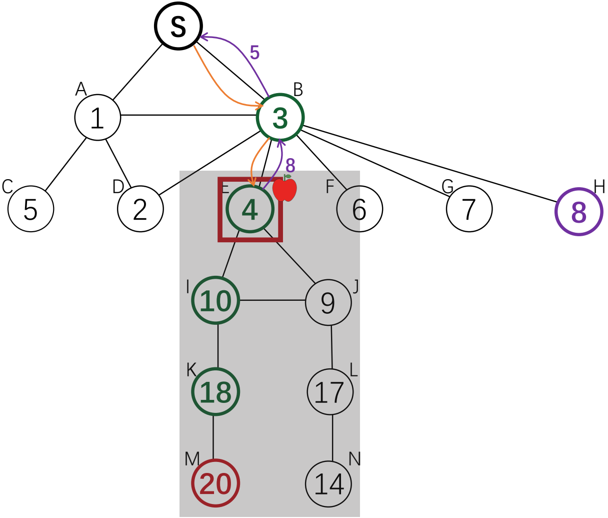

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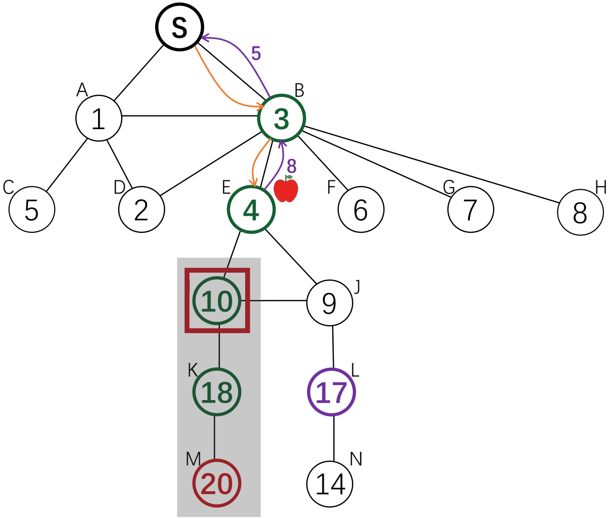

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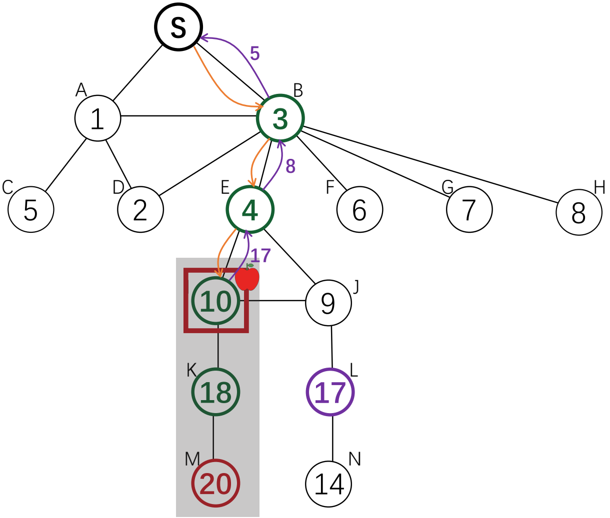

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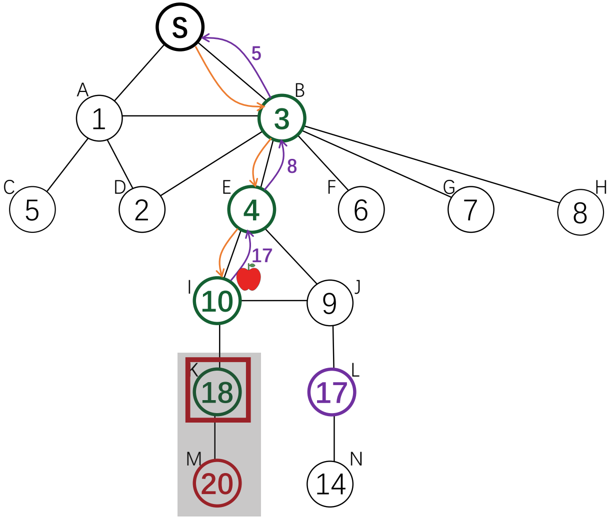

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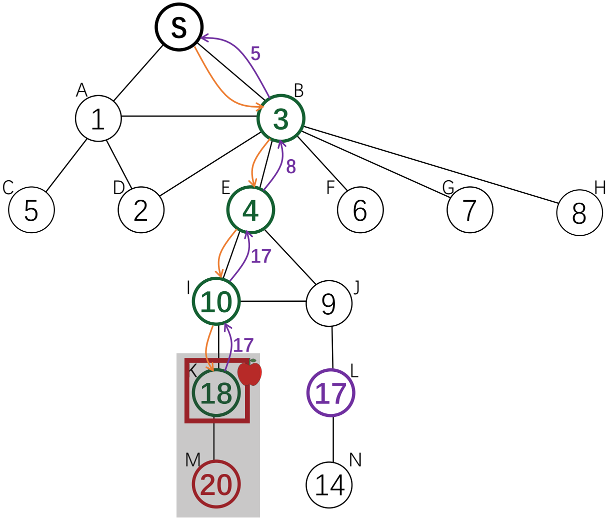

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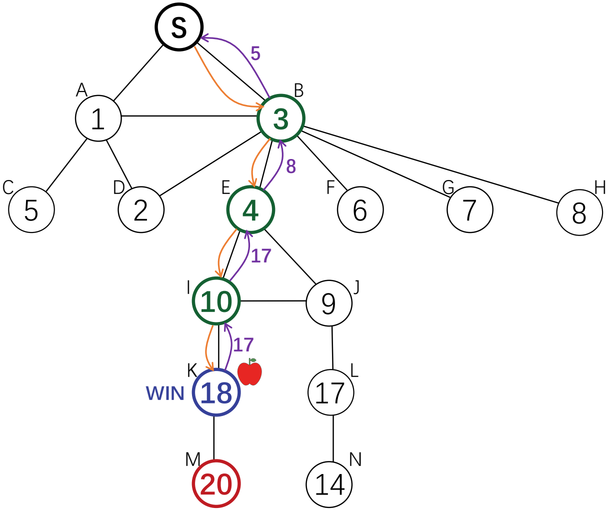
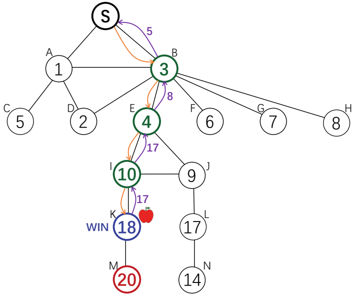


Information Diffusion Mechanism

The outcome of the Information Diffusion Mechanism:

- the item is allocated to node **K**.
- node **K** pays 17 to **I**, **I** pays 17 to **E**, **E** pays 8 to **B**, **B** pays 5 to the **seller** (a resale process).
- the utilities of **K**, **I**, **E**, **B**, **seller** are 1, 0, 9, 3, 5.



Why buyers are happy to diffuse the information?

- Buyers receive the information earlier have higher priority to win the item (K chooses before M, I chooses before K, E chooses before I, and B chooses before E).
- Diffuse the information to more buyers will potentially increase their reward (if E does not invite J, his/her utility would be 0).

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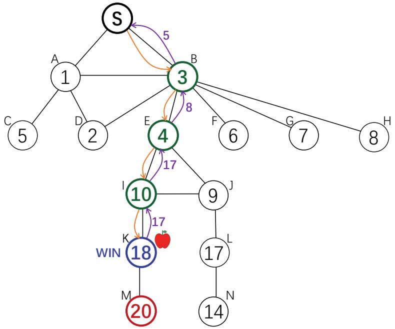

Node **K** wins the item, although M's bid is higher.

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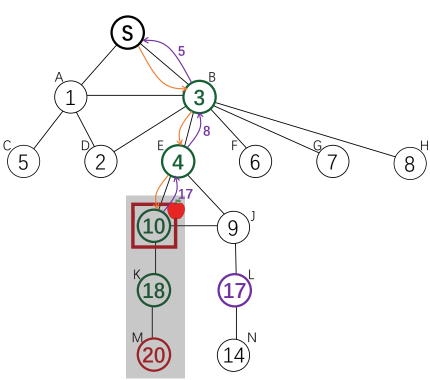
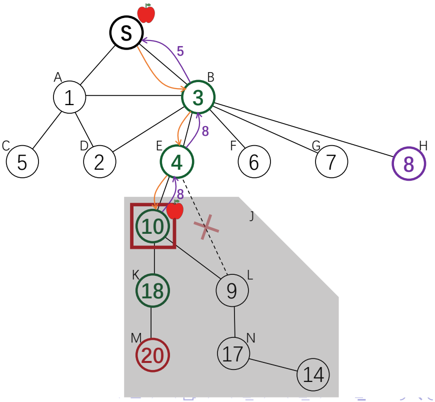
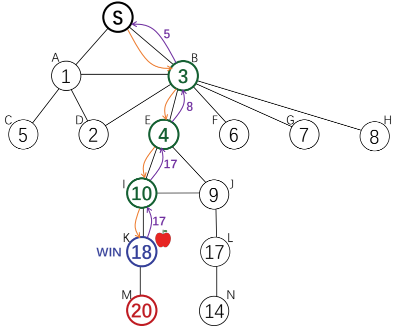




Properties of the Information Diffusion Mechanism



- Truthful: report true valuation and diffuse the sale information to all her neighbours is a dominate strategy.
- Individually Rational: no buyer will receive a negative utility to join the mechanism.
- Seller's Revenue Improved: the seller's revenue is non-negative and is $\geq$ that of the VCG without diffusion.

Outline

Diffusion Mechanisms for Combinatorial Exchanges

Challenge

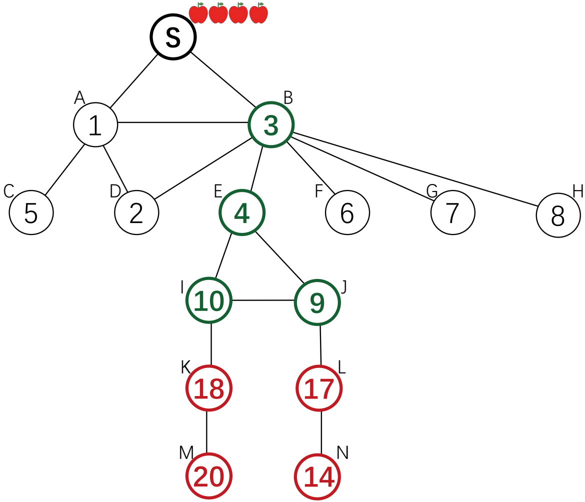
How to generalise the mechanism to combinatorial settings?

A Generalised Setting

- A seller sells $\mathcal{K} \geq 1$ homogeneous items (multi-unit supply);
- each buyer requires at most one item (single-unit demand);

Apply IDM on the Generalised Setting

Consider $\mathcal{K} = 4$:



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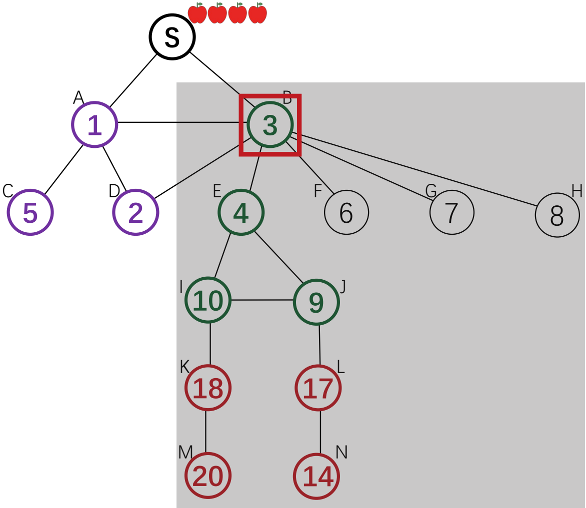

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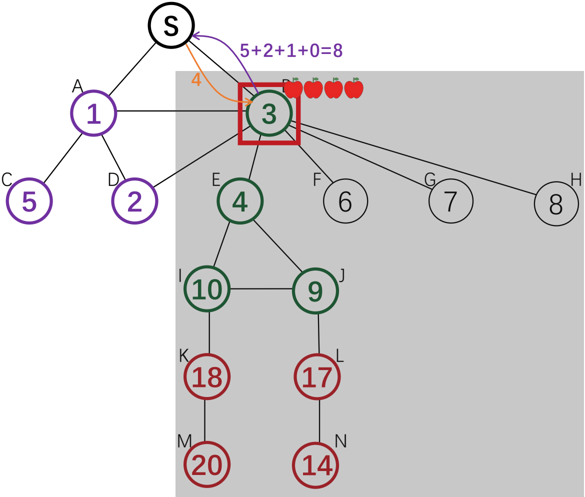

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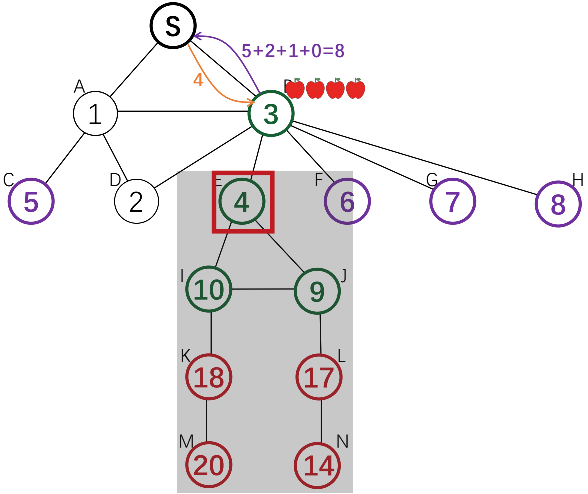

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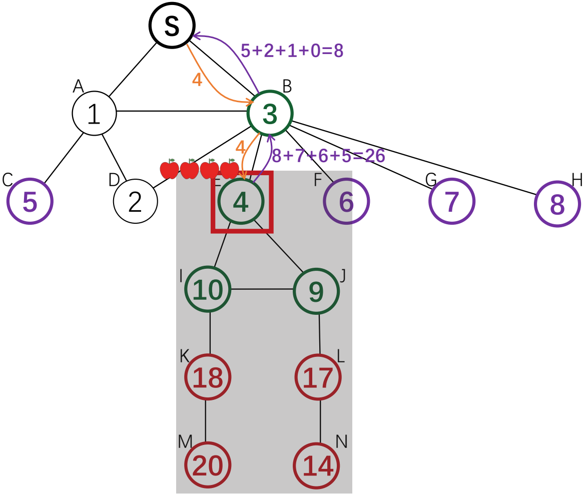

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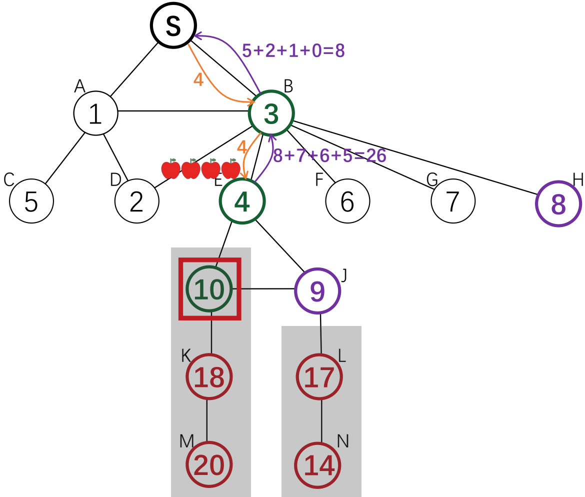

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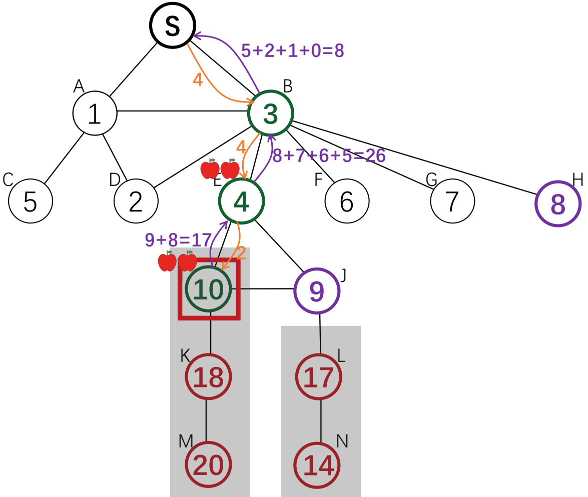

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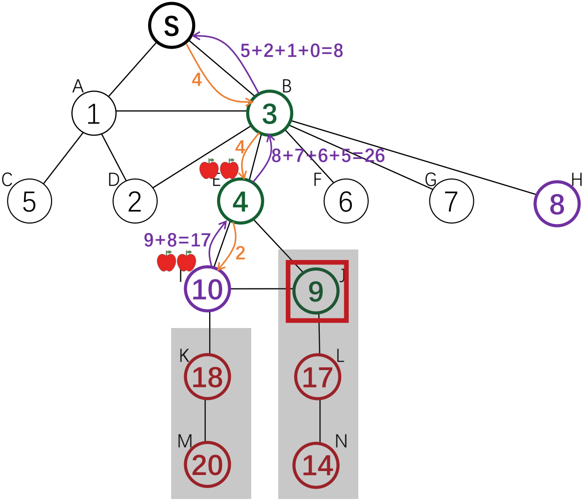

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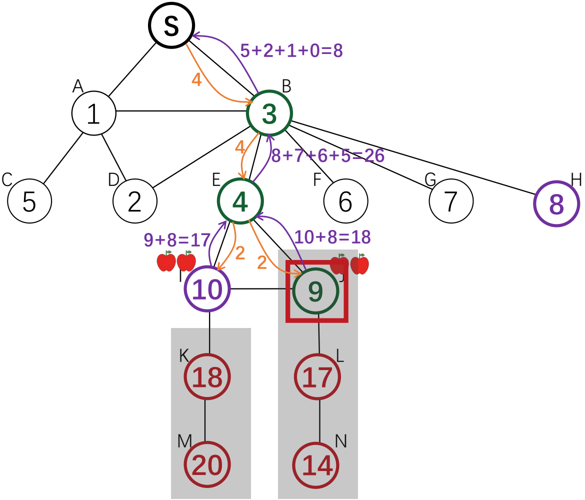
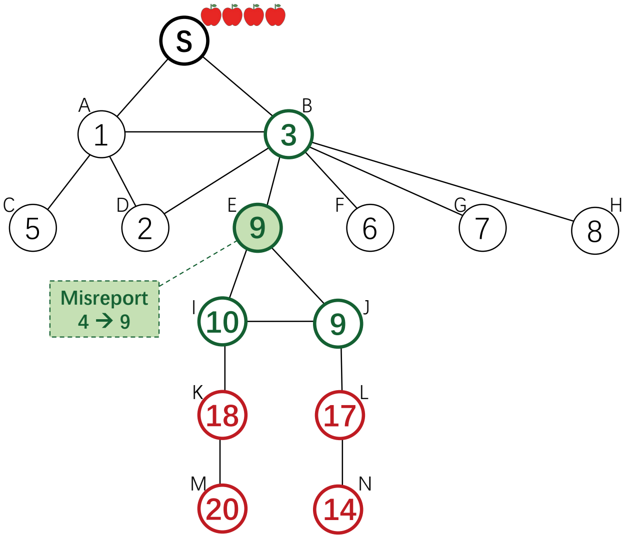


Is IDM Truthful in the Generalised Setting?



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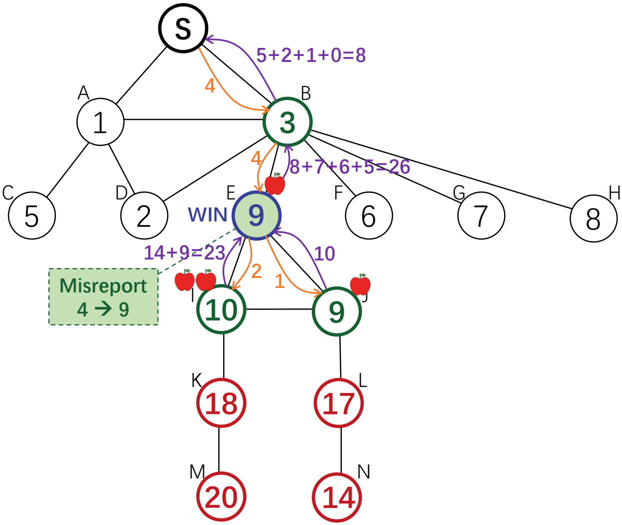

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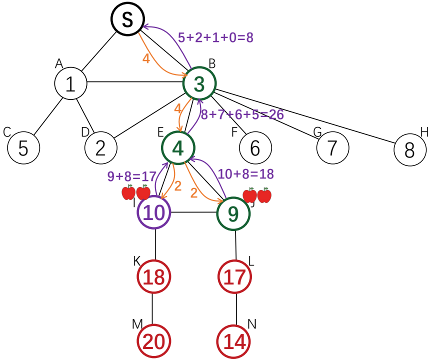
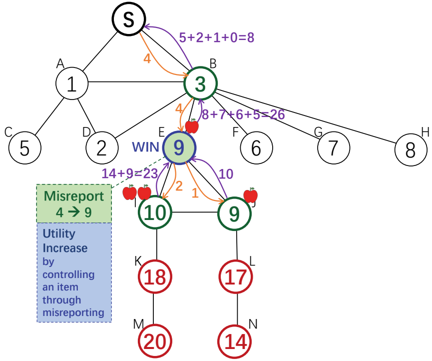


utility of E $17 + 18 - 26 = 9$



utility of E $4 + 23 + 10 - 26 = 11$

Solution to Sell Multiple Homogeneous Items

Selling Multiple Items via Social Networks [Zhao et al. AAMAS'18]

- generalises the result from [Li et al. 2017];
- buyer *i*'s reward/payment doesn't depend on the allocation of *i*'s children;
- buyers' payments are independent;

The setting:

- A seller sells $\mathcal{K} \geq 1$ homogeneous items;
- each buyer requires at most one item (single-unit demand);
- the rest is the same as [Li et al. 2017].

The Allocation Policy of the Generalisation

Buyer i receives one item if and only if

1. the top $\mathcal{K}$-highest valued children of i (and their parents, who are also i's children) do not participate
2. and i wins under the efficient allocation with their absence

given that all i's (critical) parents' allocation is determined and fixed.

The Payment Policy of the Generalisation

Buyer i's utility is the social welfare difference of the efficient allocation between

1. the top $\mathcal{K}$-highest valued children of i (and their parents, who are also i's children) do not participate (guarantees that i's payment does not depend on how many items i's children get)
2. and i (and all her children) do not participate

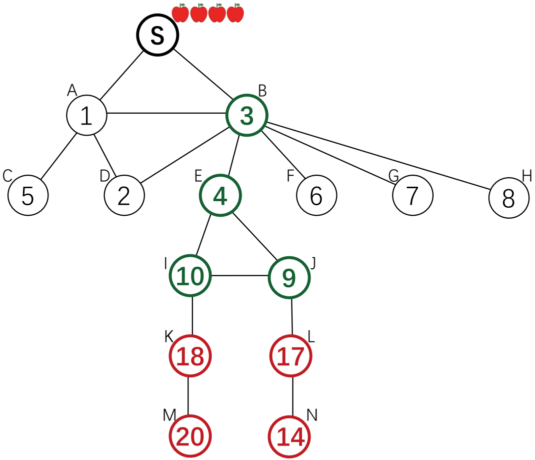
So, i's payment is:

$$\begin{cases} \mathcal{SW}_{-D_i} - (\mathcal{SW}_{-\mathcal{C}_i^{\mathcal{K}}} - v_i') & \text{if } i \in W, \\ \mathcal{SW}_{-D_i} - \mathcal{SW}_{-\mathcal{C}_i^{\mathcal{K}}} & \text{if } i \in \bigcup_{j \in W} \mathcal{P}_j(\theta') \setminus W, \\ 0 & \text{otherwise.} \end{cases}$$

where W is the set of buyers who received items.

The Generalised Diffusion Mechanism

Consider $\mathcal{K} = 4$:



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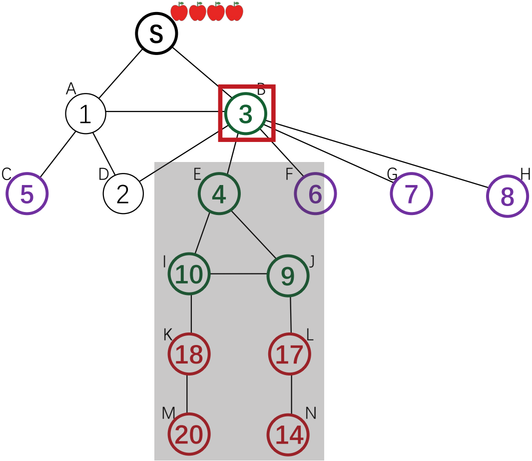

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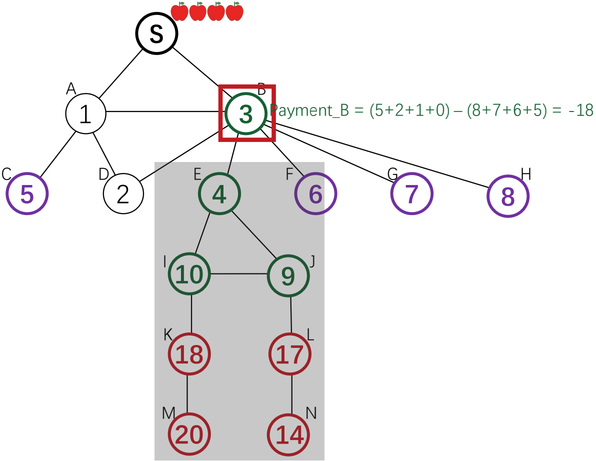

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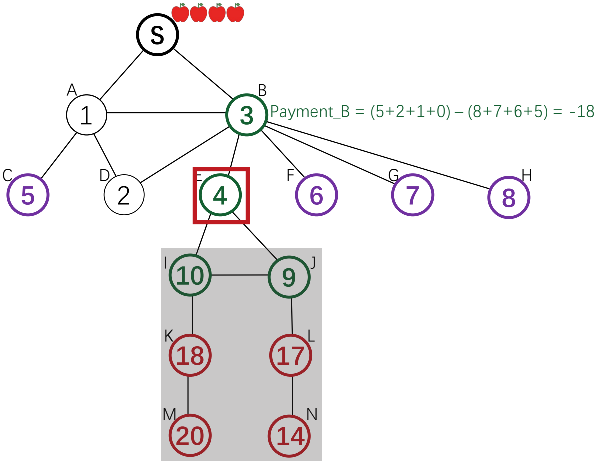

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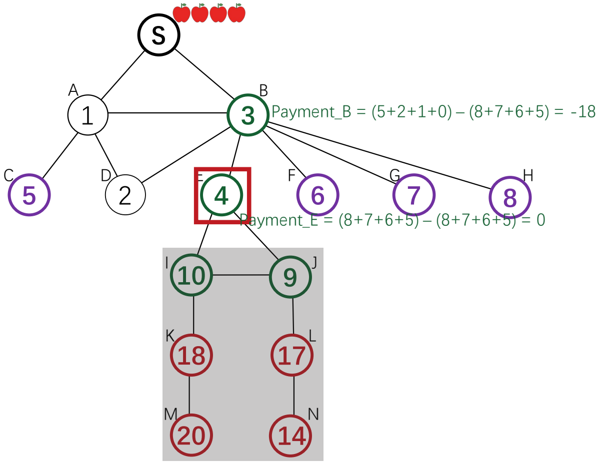

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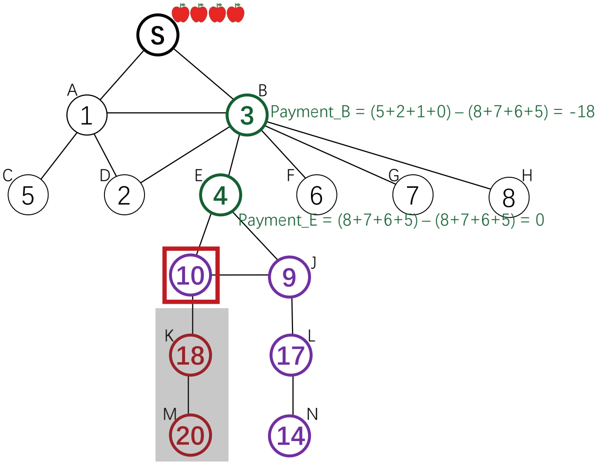

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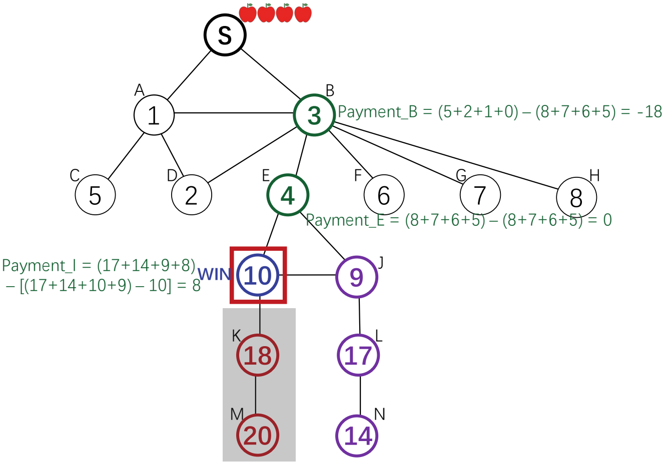

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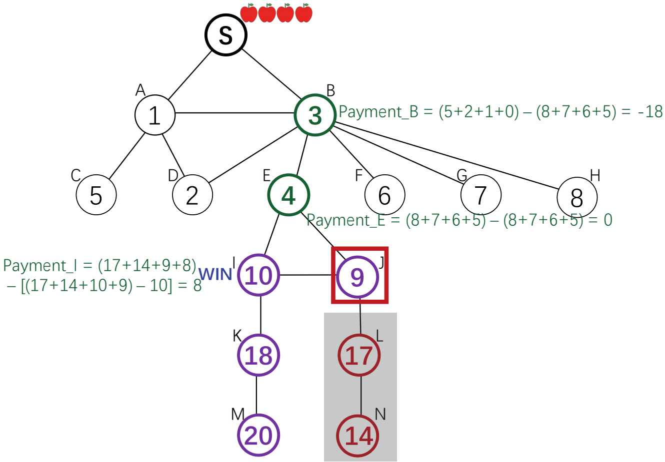

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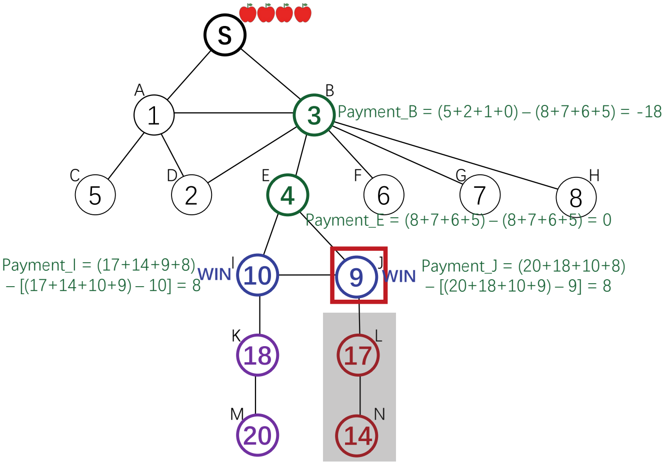

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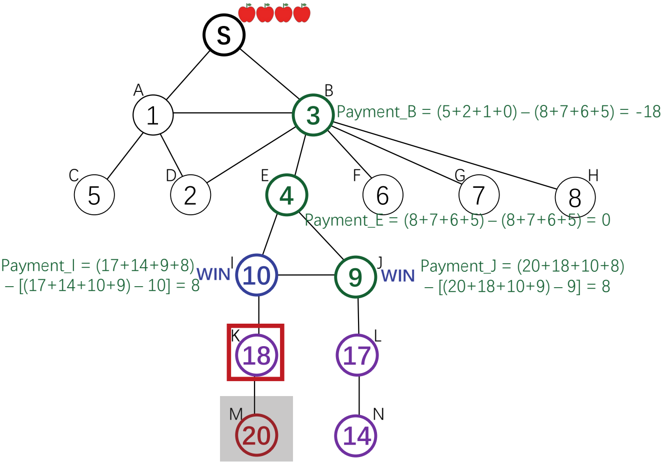

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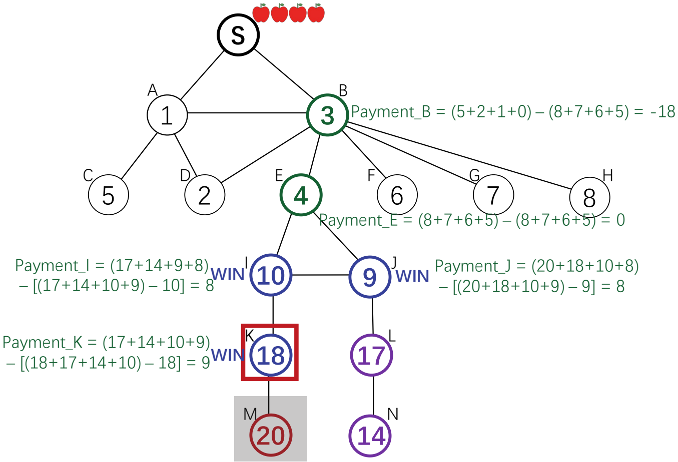

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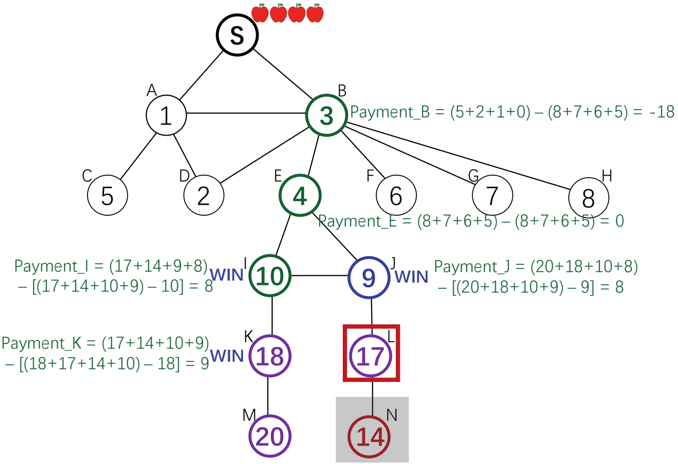

The Generalised Diffusion Mechanism

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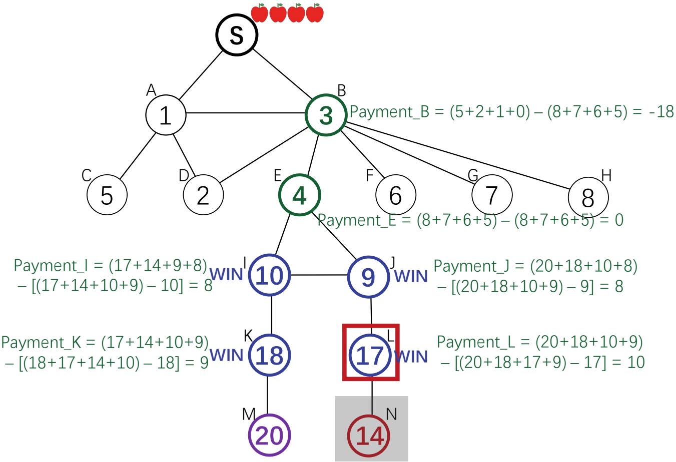

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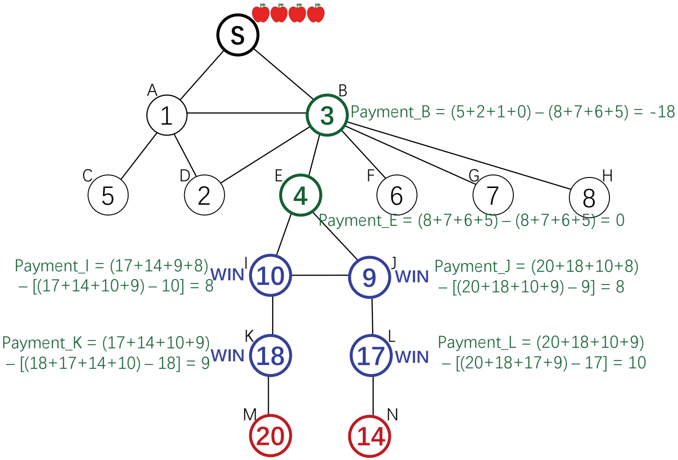

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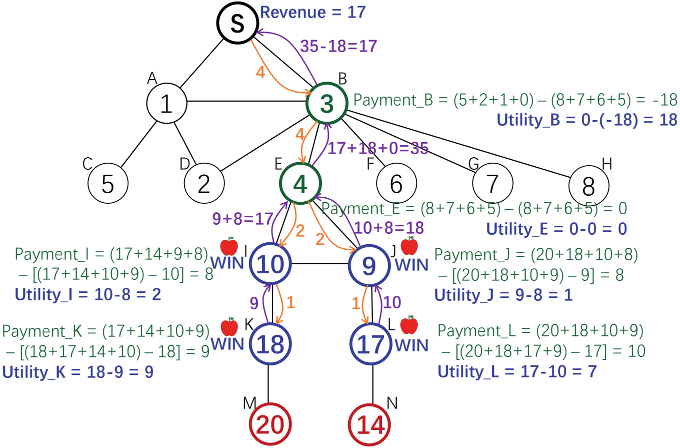
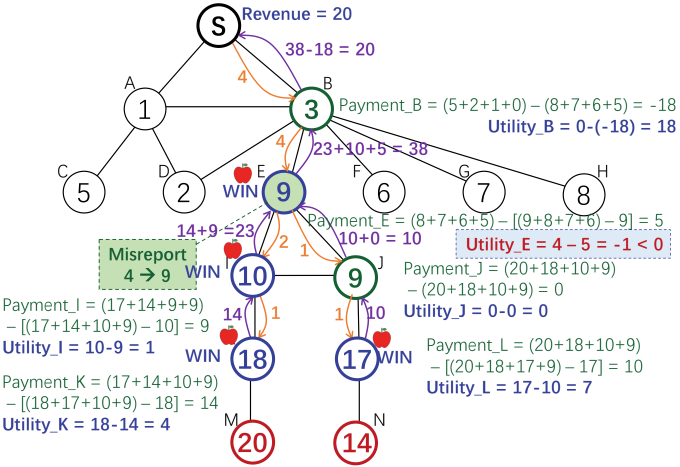


What if E misreports 9?



Properties of the Generalisation

- Truthful: report true valuation and diffuse the sale information to all her neighbours is a dominate strategy for each node.
- Individually Rational: no node will receive a negative utility to join the mechanism.
- Seller's Revenue Improved: the seller's revenue is non-negative and is $\geq$ that of the VCG without diffusion.

Outline

Other Way to Sell Multiple Homogeneous Items?

The Main Idea

The probability for a participant to win the item increases if he/she invites other participants.

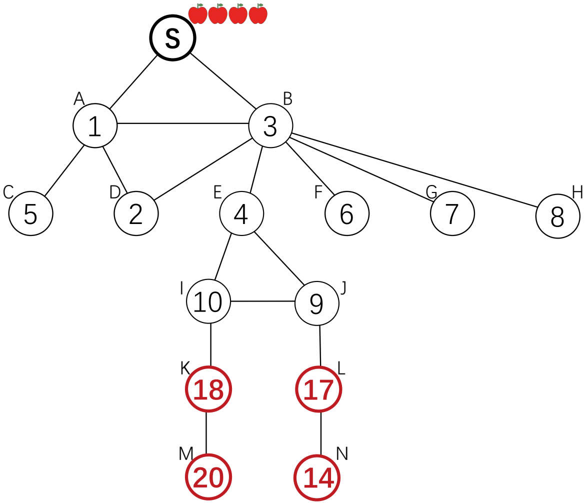
Distance-Based Network Auction Mechanism for Multi-unit, Unit-demand Buyers (DNA-MU)

Strategy-Proof and Non-Wasteful Multi-Unit Auction via Social Network [Kawasaki et al. AAAI'20]

- Order every buyer based on their shortest distance to the seller, buyers with shorter distance has a higher priority.
- Initialize $\mathcal{K}' = \mathcal{K}$. Traverse all buyers based on their priority, if $v_i' \geq v_{-D_i}^{\mathcal{K}'}$ (the $\mathcal{K}'$-th largest value of the buyers in $-D_i$ who didn't receive item yet), allocate one item to i, i pays $v_{-D_i}^{\mathcal{K}'}$, $K' = K' - 1$.

DNA-MU

Consider $\mathcal{K} = 4$:



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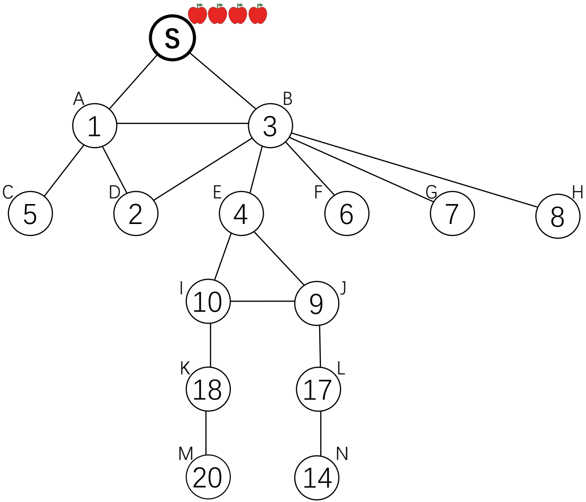

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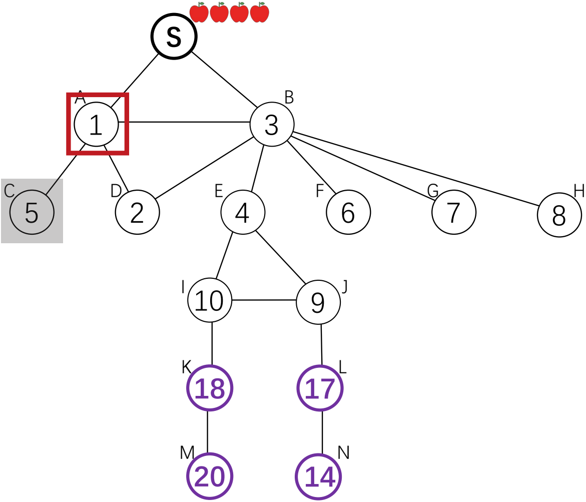

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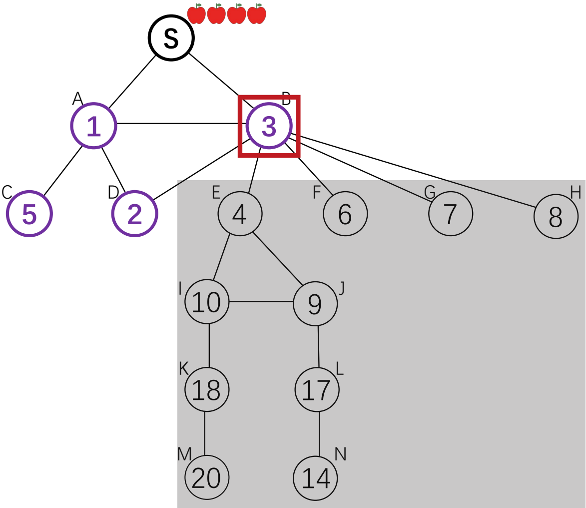

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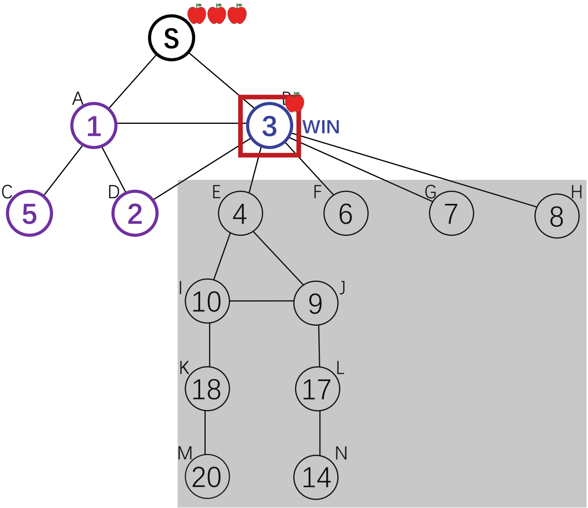

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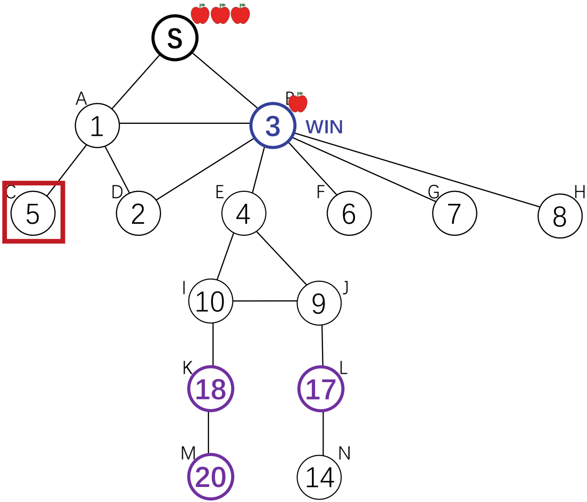

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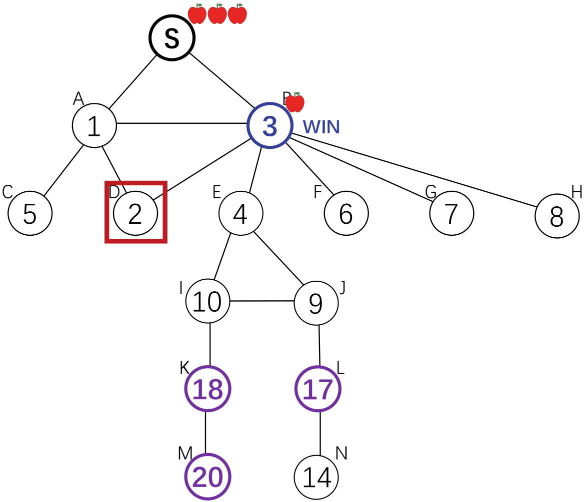

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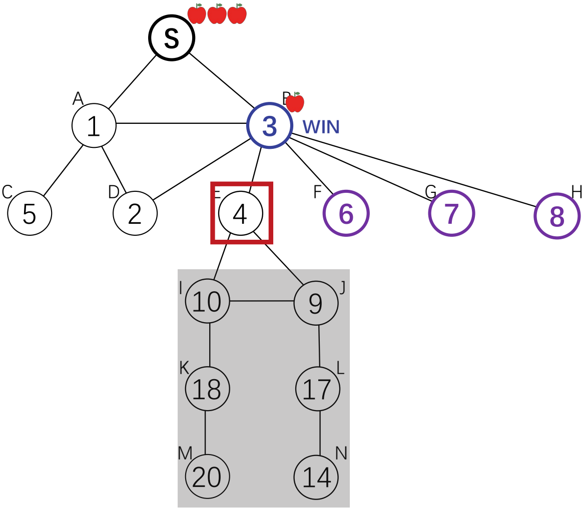

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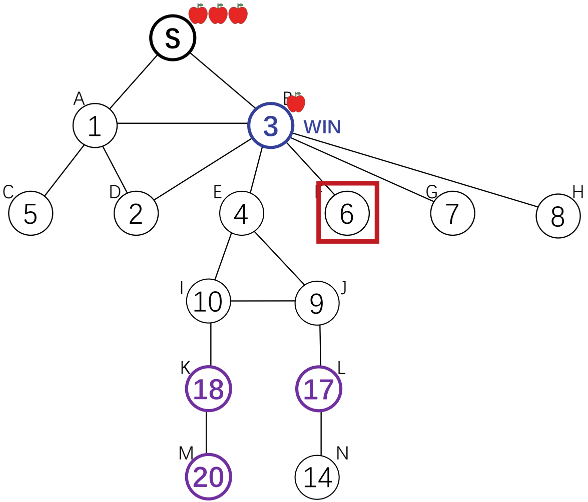

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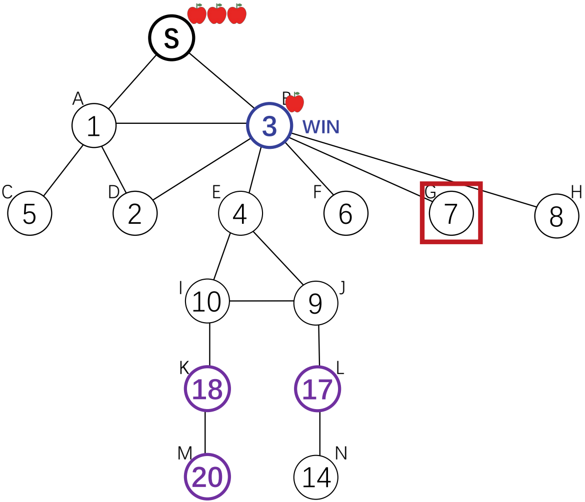

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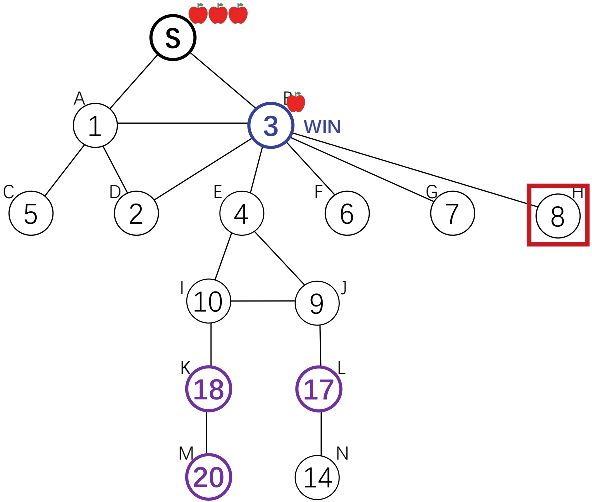

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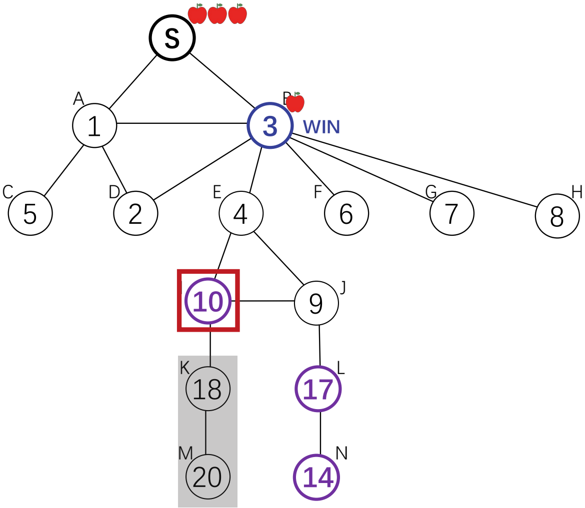

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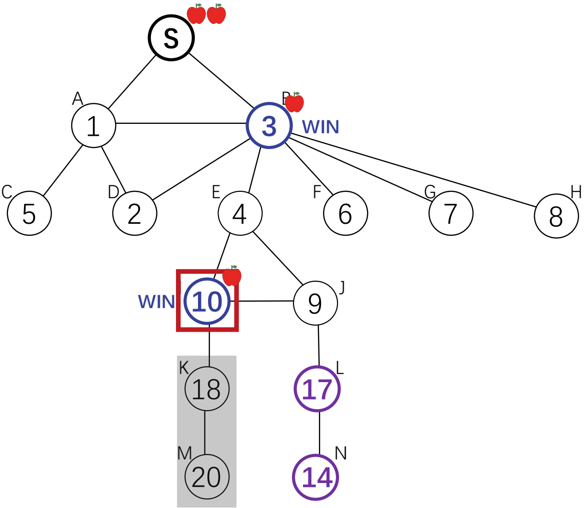

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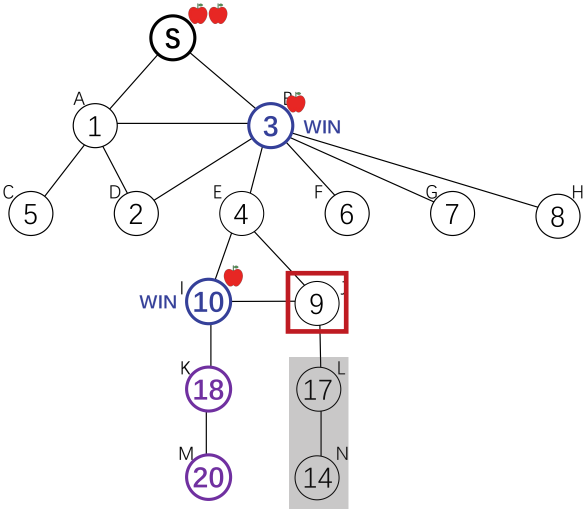

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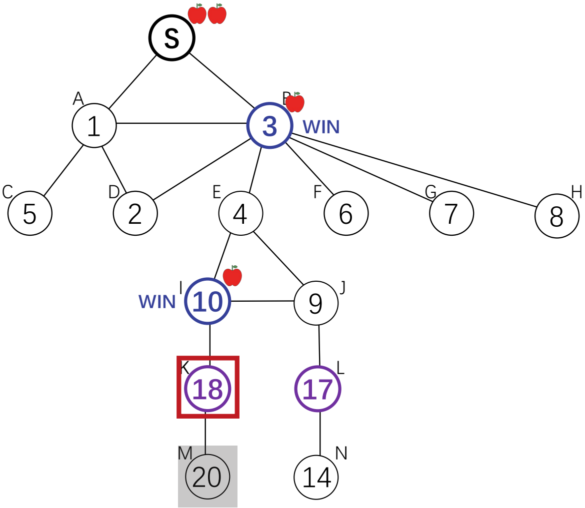

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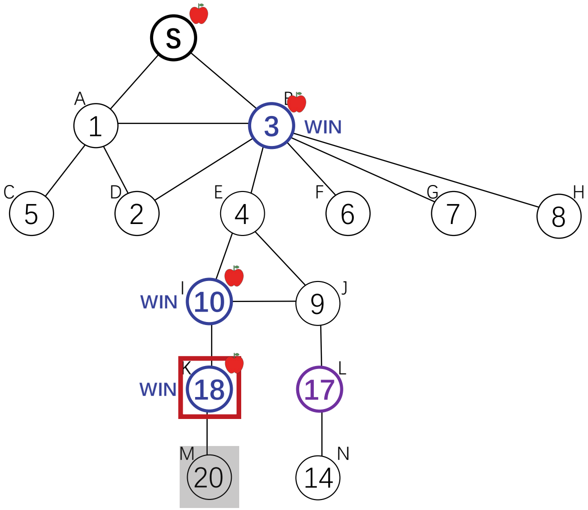

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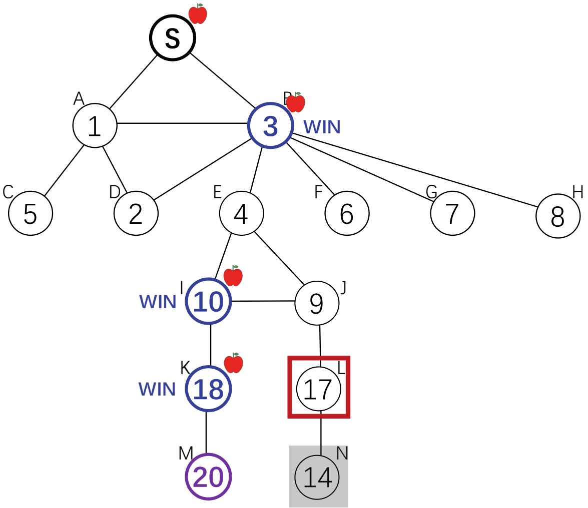

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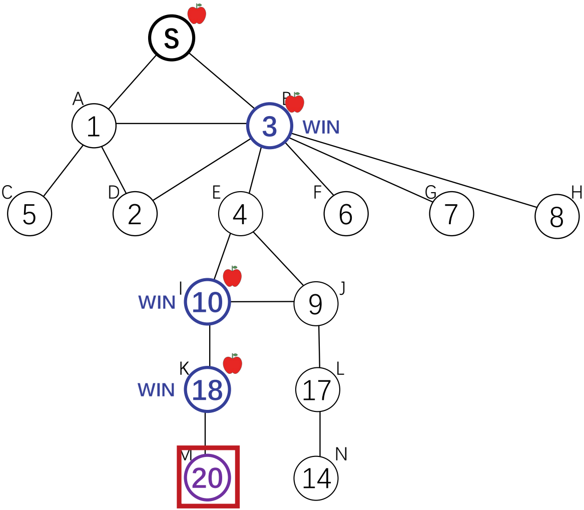

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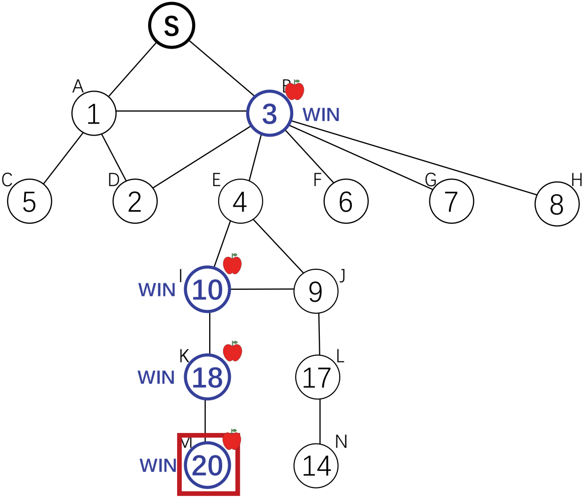

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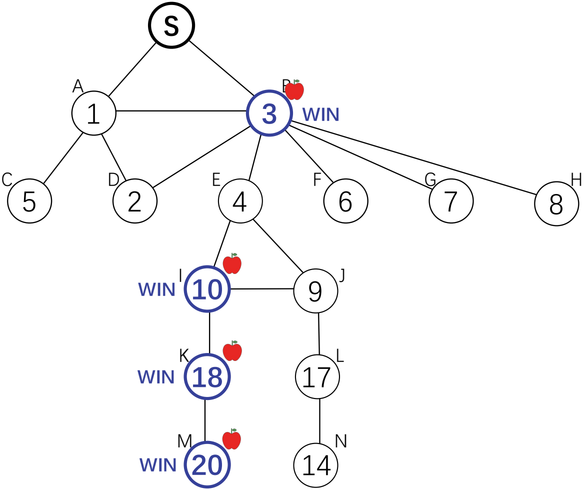
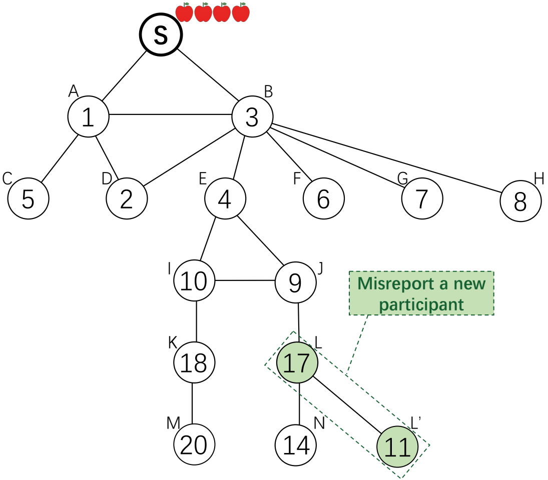


Participants can raise his/her own probability to win by misreporting a new invitee.



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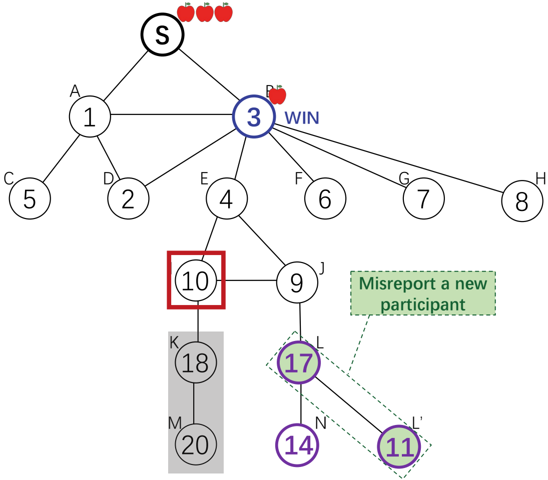

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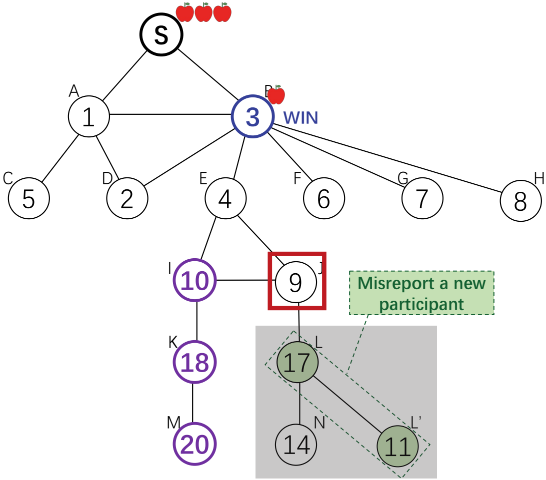

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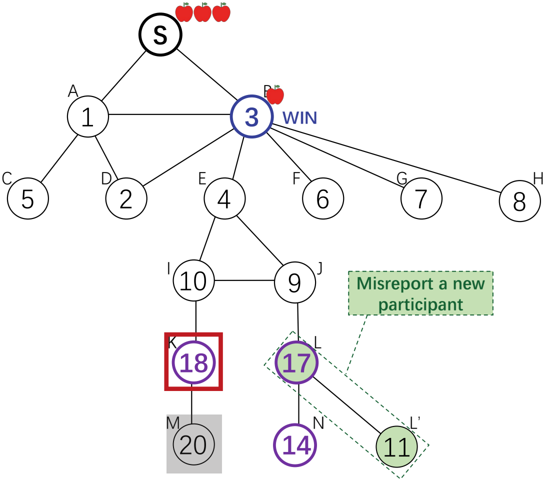

Distance-Based Mechanism

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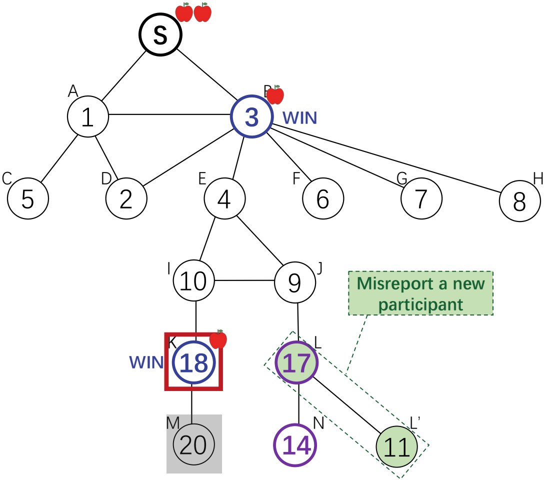

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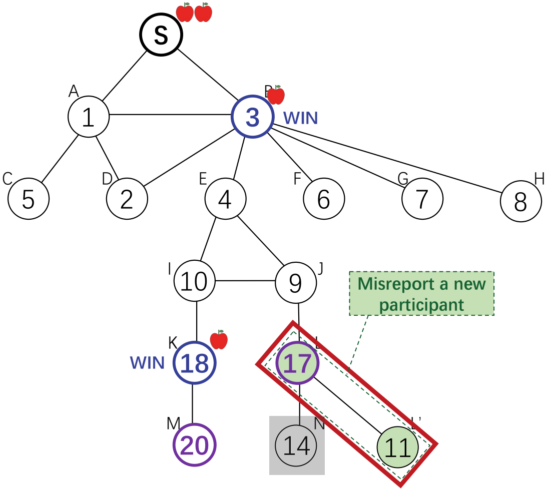

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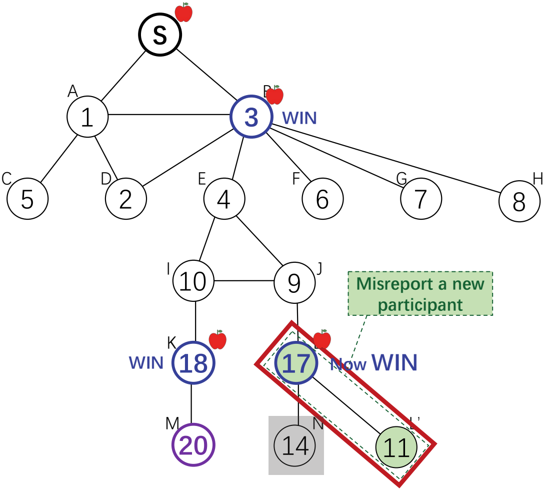

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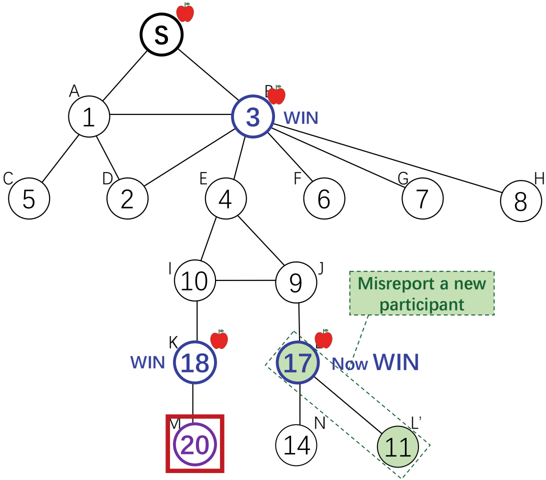

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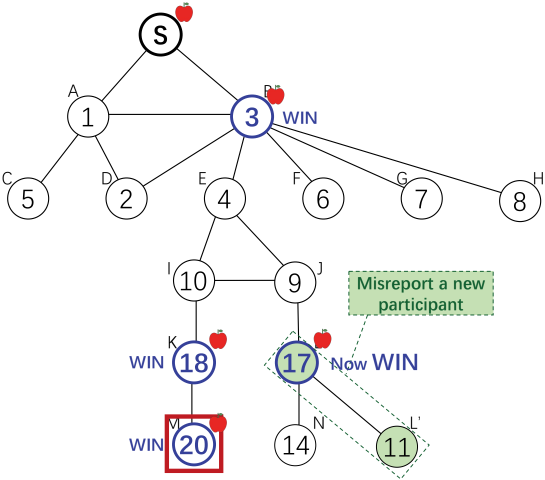

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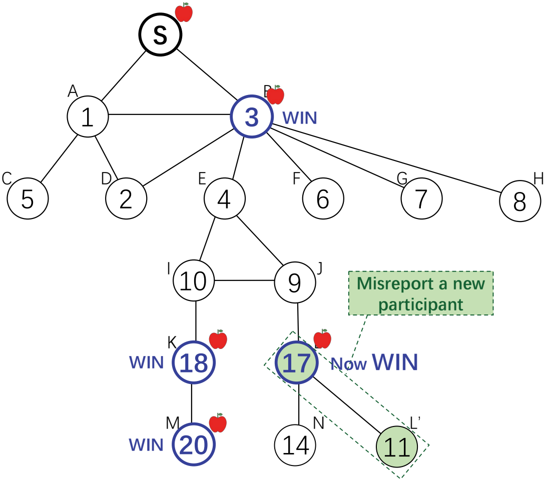
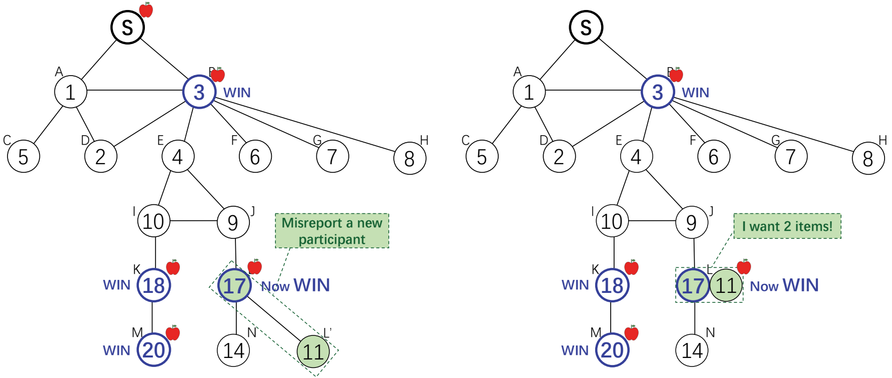


Participants can raise his/her own probability to win by misreporting a new invitee.

Inviting a new invitee works the same as reporting 2 prices.



The mechanism is not IC if buyers require more than one item.

From Unit-demand to Multi-unit-demand

The Main Idea

- Buyers' utility will increase if they invite more buyers
- All potential winners' allocations/payments should be independent with each other

Ref: Liu et al. 2022: Diffusion Multi-unit Auctions with Diminishing Marginal Utility Buyers

Who are the potential winners?

Who are the potential winners?

- The buyers who have the top $\mathcal{K}$ highest valuations
- The buyers who invited the above buyers

How to guarantee to remove all potential winners?

1. if we remove all buyers who have child, then we have removed the second group for sure.
2. then for the remaining buyers, we remove the buyers with the top $\mathcal{K}$ highest valuations, to guarantee that the first group is removed.

Possible way to manipulate?

- a buyer has child may misreport no-child, by doing so, less buyers will be removed, so the buyer may win an item to gain

Other Interesting Extensions

1. a characterization of diffusion auctions for single-unit
   - Li et al. IJCAI'20: Incentive-Compatible Diffusion Auctions
2. for fair reward sharing (single-unit case)
   - Zhang et al. AAMAS'20: Redistribution Mechanism on Networks
3. for non-profit (single-unit case)
   - Zhang et al. ECAI'20: Incentivize Diffusion with Fair Rewards
4. consider cost for diffusion (single-unit case)
   - Li et al. IJCAI'19: Diffusion and Auction on Graphs

Open Questions

- more challenging for more general settings
  - truthful diffusion mechanisms for heterogeneous items, even just for two items
  - revenue monotonicity, false-name proofness and truthfulness come together?
- when there is a diffusion cost
  - how to guarantee each diffusion is beneficial (budget balance)?
- privacy concern and the seller's strategies
  - the owner of the mechanism discoveries the whole network and she may cheat as well!
- Sybil-attack (false-name manipulations)
  - diffusion incentive conflicts with Sybil-attack if we cannot verify their identities
- many more...

Outline

Outline

Coalitional/Cooperative Games

- A set of agents N.
- Each subset of agents(coalition) $S \subseteq N$ cooperate together can generate some value $v(S) \in R$. Assume $v(\emptyset) = 0$. N is called grand coalition. $v : 2^N \to R$ is called the characteristic function of the game. v is often assumed to be monotonic: $S \subseteq T \to v(S) \leq v(T)$.
- The possible outcomes of the game is defined by $V(S) = \{x \in R^S : \sum_{i \in S} x_i \leq v(S)\}$.

An Example

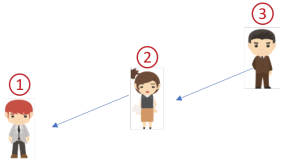
- Three agents $\{1, 2, 3\}$.
- $v(\{1\}) = v(\{2\}) = 10, v(\{3\}) = 1;$
  $v(\{1, 2\}) = 20, v(\{1, 3\}) = v(\{2, 3\}) = 12;$
  $v(\{1, 2, 3\}) = 22.$

The Shapley Value: a classic value distribution

Given a coalitional game (N, v), the Shapley value of each player i is defined as:

$$\phi_i(v) = \sum_{S \subset N \setminus \{i\}} \frac{|S|!(n - |S| - 1)!}{n!} (v(S \cup \{i\}) - v(S))$$

where $v(S \cup \{i\}) - v(S)$ is the marginal contribution for i to join S, denoted as c_i



- Agent 1 alone,
  $c_1 = v(\{1\}) - v(\emptyset) = 10$
- Agent 2 joins,
  $c_2 = v(\{1, 2\}) - v(\{1\}) = 10$
- Agent 3 joins,
  $c_3 = v(\{1, 2, 3\}) - v(\{1, 2\}) = 2$

The Shapley Value: a classic value distribution

For the early example:

- all the permutations and their marginal contributions are:

Permutation	c_1	c_2	c_3
1,2,3	10	10	2
1,3,2	10	10	2
2,3,1	10	10	2
2,1,3	10	10	2
3,1,2	11	10	1
3,2,1	10	11	1

- their Shapley value is $\phi_i = \frac{\sum c_i}{6}$: $\phi_1 = \frac{61}{6}, \phi_2 = \frac{61}{6}, \phi_3 = \frac{10}{6}$

How to attract more participants to join them?

- We want to incentivize the existing participants to invite their neighbors to join (assume there is a network).
- Assum agent 1 invites her neighbor $1'$ who is equivalent to agent 1 (i.e. $v(\{1\} \cup S) = v(\{1'\} \cup S) = v(\{1, 1'\} \cup S)$)

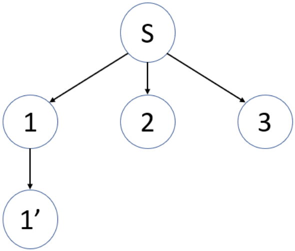
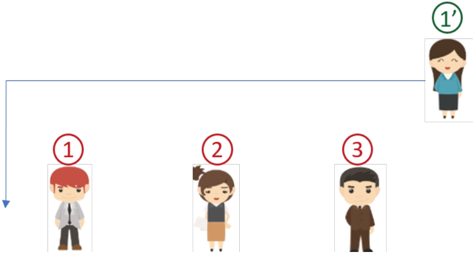


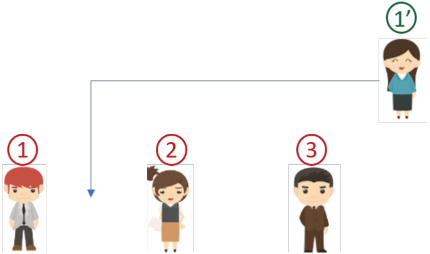
Figure: s is the game host/sponsor, s can invite $1, 2, 3$ and 1 can invite $1'$

Agent $1'$ will reduce the Shapley of agent 1



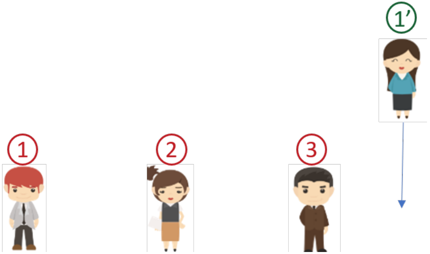
- Without agent $1'$:$c_1 = 10$, $c_2 = 10$, $c_3 = 2$
- With agent $1'$:$c'_{1'} = 10$, $c'_1 = 0$, $c'_2 = 10$, $c'_3 = 2$

Agent $1'$ will reduce the Shapley of agent 1



- Without agent $1'$: $c_1 = 10$, $c_2 = 10$, $c_3 = 2$
- With agent $1'$: $c_1' = 10$, $c_{1'}' = 0$, $c_2' = 10$, $c_3' = 2$

Agent $1'$ will reduce the Shapley of agent 1



- Without agent $1'$: $c_1 = 10$, $c_2 = 10$, $c_3 = 2$
- With agent $1'$: $c_1' = 10$, $c_2' = 10$, $c_3' = 2$, $c_{1'}' = 0$

Agent $1'$ will reduce the Shapley of agent 1

- Similarly, we can get all the permutations and their marginal contributions.

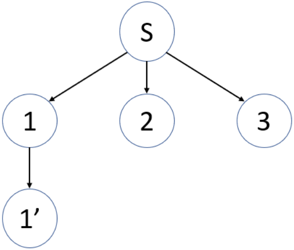
Permutation	c_1	c_2	c_3	$c_{1'}$
$1,2,3,1'$	$10 \to 10$	10	2	0
$1,3,2,1'$	$10 \to 10$	10	2	0
...	...	...	...	...
$3,1',1,2$	$11 \to 0$	10	1	11
$1',3,2,1$	$10 \to 0$	10	2	10

- For the permutation where agent $1'$ is before 1, $c_1 = 0$ since agent $1'$ performs the same as 1.

Agent $1'$ will reduce the Shapley of agent 1

Challenge

Directly applying Shapley value cannot incentivize players to invite others.



Outline

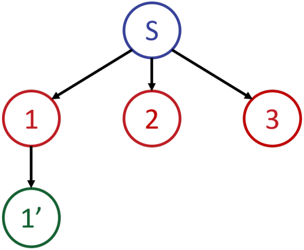
A Layer-based Solution

Main Idea

- Divide the agents into layers in terms of distance
- Lower layers always join the game earlier than higher layers

Ref: Zhang et al. AAMAS'20: Collaborative Data Acquisition.

Layered Shapley Value



- Recalling the previous example, agents 1,2,3 are in the first layer while agent $1'$ is in the second layer
- We only allow permutations where $1'$ is after the set $\{1, 2, 3\}$ (e.g. 1,3,2,$1'$)

Layered Shapley Value

1′ 1 2 3	1 1′ 2 3	1 2 1′ 3	1 2 3 1′
1′ 1 3 2	1 1′ 3 2	1 3 1′ 2	1 3 2 1′
1′ 2 1 3	2 1′ 1 3	2 1 1′ 3	2 1 3 1′
1′ 2 3 1	2 1′ 3 1	2 3 1′ 1	2 3 1 1′
1′ 3 1 2	3 1′ 1 2	3 1 1′ 2	3 1 2 1′
1′ 3 2 1	3 1′ 2 1	3 2 1′ 1	3 2 1 1′

- Thus, only the permutations in the blue dotted box are considered in the marginal contribution computation.

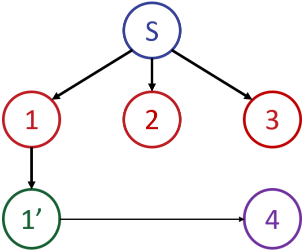
Layered Shapley Value

- Then their marginal contributions for the remaining permutations are the following, where the contribution for agent $1'$ is 0 now, by doing so, agent 1 is incentivized to invite agent $1'$, as $1'$ will never take his marginal contribution.

Permutation	c_1	c_2	c_3	$c_{1'}$
$1,2,3,1'$	10	10	2	0
$1,3,2,1'$	10	10	2	0
$2,1,3,1'$	10	10	2	0
$2,3,1,1'$	10	2	10	0
$3,1,2,1'$	11	10	1	0
$3,2,1,1'$	10	11	1	0

- However, why agents $2, 3$ should come before agent $1'$?

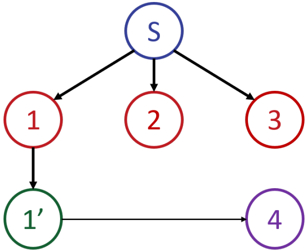
A General Solution: Permission Shapley Value



The setting ($1'$ has another neighbor 4):

- In the previous example, suppose agent $1'$ invites agent 4.
- $v(\{1\}) = v(\{2\}) = 10.$
  $v(\{3\}) = 1;$
  $v(\{1,2\}) = 20;$
  $v(\{1,3\}) = v(\{2,3\}) = 12;$
  $v(\{1,2,3\}) = 22.$
- $v(\{1'\} \cup S) = v(\{1\} \cup S) = v(\{1,1'\} \cup S)$
- $v(\{4\} \cup S) = v(S) + 100$

Permission Structure



Permission structure:

- A permission structure on N is an asymmetric mapping p:$N \to 2^N$, i.e., $j \in p(i)$ implies that $i \notin p(j)$
- $p(i)$: the set of players who invited i into the coalition.
- In the example, $p(1') = \{1\}, p(4) = \{1'\}$.

Definition

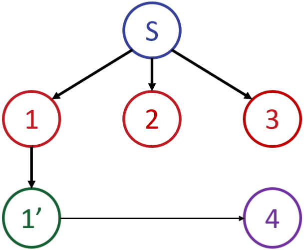
Autonomous A coalition $S \subseteq N$ is autonomous in a permission structure p if for all $i \in S, p(i) \subseteq S$.

- where $\{1, 2, 1'\}$ is autonomous, but $\{1, 2, 4\}$ is not as $p(4) = \{1'\} \not\subseteq \{1, 2, 4\}$.

Permission Structure + Shapley Value

Largest Autonomous Part

Let p be a permission structure on N. Then the largest automonous part of a coalition $S \subset N$ is defined by $\alpha(S) = \cup\{T | T \subseteq S \textit{ and } T \in A_p\}$ where A_p is the collection of all autonomous coalitions under p.



- define $v^p(S) = v(\alpha(S))$, e.g. $v^p(\{1,2,3,4\}) = v(\{1,2,3\}) = 22$ since $\alpha(\{1,2,3,4\}) = \{1,2,3\}$
- compute the Shapley value based on $v^p(S)$

Permission Structure + Shapley Value

- consider permutation {1,2,4,3,1′} as an example.

S	$\alpha(S)$	$v(S)$	$v^p(S)$
1	1	10	10
1,2	1,2	20	20
1,2,4	1,2	120	20
1,2,4,3	1,2,3	122	22
1,2,4,3,1′	1,2,4,3,1′	122	122

- $\phi_1 = \frac{61}{6} + \frac{100}{3}$, $\phi_2 = \frac{61}{6}$, $\phi_3 = \frac{10}{6}$, $\phi_{1'} = \frac{100}{3}$, $\phi_4 = \frac{100}{3}$
- *Ref: Zhang and Zhao AAMAS'22: Incentives to Invite Others to Form Larger Coalitions.*

Open Questions

- characterize all mechanisms to incentivize invitations
- when the game is not monotonic (marginal contribution of some players can be negative)
- what is a core in the network setting?
- how to prevent Sybil-attack?

Outline

House Allocation (One-sided Matching)

The setting:

- There are n agents, and each agent has a house/item.
- Each agent has a strict preference over all the houses/items.

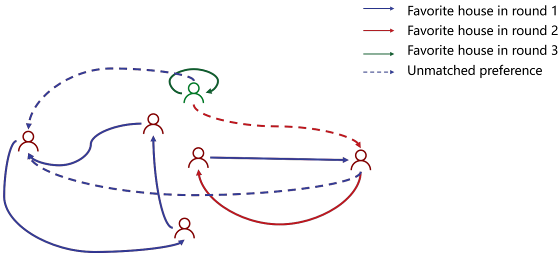
Outline

The Top Trading Cycle (TTC) Mechanism

Top Trading Cycle (TTC):

1. Ask each agent to indicate her favorite house.
2. Draw an arrow from each agent i to the agent, denoted $Top(i)$, who holds the favorite house of i.
3. Note that there must be at least one cycle in the graph (this might be a cycle of length 1, if some agent i currently holds his own top house). Implement the trade indicated by this cycle (i.e., reallocate each house to the agent pointing to it), and remove all the involved agents from the game.
4. If there are remaining agents, go back to step 1.

The Top Trading Cycle (TTC) Mechanism



Properties

- Truthful: dominant strategy for agents to report true preferences.
- Stable: no subset of agents can improve their allocation.

House Allocation in Network

The setting:

- There are *n* agents, and each agent has a house.
- Each agent has a strict preference over all the houses.
- Each agent has a set of neighbors, and doesn't know the others.
- Initially only a subset of agents are in the game.

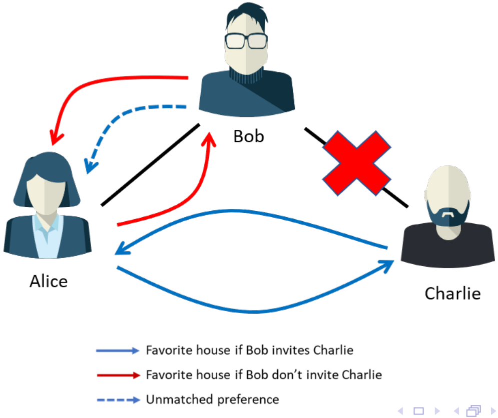
The goal:

- Report preferences truthfully
- Invite all their neighbors to join the matching

House Allocation in Network

Can we directly apply TTC in the network setting to incentivize invitation?

- No, an invitee may compete with the inviter



How to incentivize an agent to invite others?

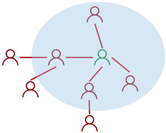
To incentivize an agent to invite others:

- The agent's allocation is not getting worse after her invitations.

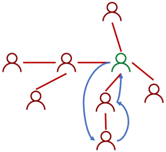
One Solution: Add Restrictions on TTC

Restrict the range each agent can exchange with:

- **Swap With Neighbors**: only allow each agent to exchange within her neighbors, no competition between invitees and inviters.



- **Swap With Children**: if an agent wants to trade with her neighbors' neighbor, she must get the permission of her neighbor.



Restricted TTC: Swap With Neighbors

1. Construct a directed graph by the preference of each agent: each agent points to her favorite item among herself and her neighbors remaining in the matching.
2. There is at least one cycle. For each cycle, allocate the item to the agent who points to it and remove the cycle.
3. Repeat the process until there is no agent left.

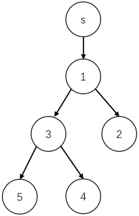
Restricted TTC: Swap With Children in Trees

1. Construct a directed graph by the preference of each agent: each agent points to her favorite item among herself, her neighbors, and her descendants remaining in the matching.
2. There is at least one cycle. For each cycle, allocate the item to the agent who points to it and remove the cycle.
3. Repeat the process until there is no agent left.

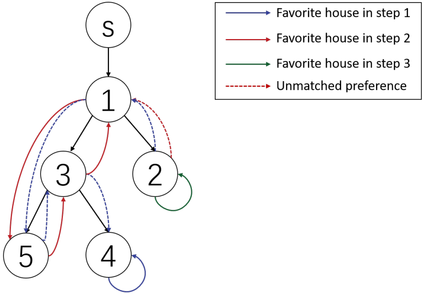
Refs:

- *Zheng et al. 2020: Barter Exchange via Friends' Friends.*
- *Kawasaki et al. AAMAS'21: Mechanism Design for Housing Markets over Social Networks.*

An Example of Swap With Children in Trees



$\succ_1: h_5 \succ_1 h_2 \succ_1 h_3 \succ_1 h_4 \succ_1 h_1 \succ_1 \dots$
$\succ_2: h_5 \succ_2 h_1 \succ_2 h_2 \succ_2 \dots$
$\succ_3: h_4 \succ_3 h_2 \succ_3 h_1 \succ_3 h_3 \succ_3 h_5 \succ_3 \dots$
$\succ_4: h_5 \succ_4 h_4 \succ_4 \dots$
$\succ_5: h_2 \succ_5 h_3 \succ_5 h_5 \succ_5 \dots$



Properties of the Restricted TTC

Properties

- Truthfully report their preferences
- Invite all their neighbours
- Not stable in general

Problem

- Too restricted allocation space, invitation does not really improve their matching
- Only work in restricted networks, cannot be generalized.

Outline

A Better Solution: Leave and Share

The intuition:

- The Swap With Neighbors mechanism allows agents to receive items from their neighbors only. This guarantees that inviters are not worse-off.
- However, when a group of agents are matched/left, they don't care the rest, thus their remaining neighbors can match with each other.

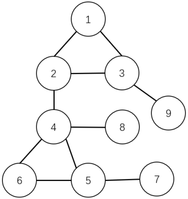
A Better Solution: Leave and Share

The Leave and Share mechanism:

- Use Swap With Neighbors as base.
- Choose a random order to decide which agent to be considered/matched first (not any cycle as in the original Swap With Neighbors).
- When a group of agents are matched/left, their remaining neighbors become neighbors of each other (dynamically update agents' neighbor sets)

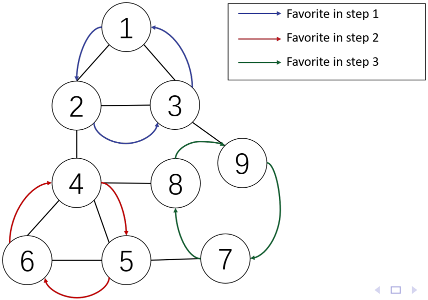
Ref: *Yang et al. 2022: One-Sided Matching with Permission.*

An Example of Leave and Share



$\succ_1$: $h_2 \succ_1 h_4 \succ_1 h_3 \succ_1 h_1 \succ_1 \ldots$
$\succ_2$: $h_3 \succ_2 h_4 \succ_2 h_5 \succ_2 h_2 \succ_2 \ldots$
$\succ_3$: $h_8 \succ_3 h_4 \succ_3 h_1 \succ_3 h_3 \succ_3 \ldots$
$\succ_4$: $h_1 \succ_4 h_5 \succ_4 h_3 \succ_4 h_4 \succ_4 \ldots$
$\succ_5$: $h_2 \succ_5 h_6 \succ_5 h_9 \succ_5 h_5 \succ_5 \ldots$
$\succ_6$: $h_4 \succ_6 h_1 \succ_6 h_8 \succ_6 h_6 \succ_6 \ldots$
$\succ_7$: $h_8 \succ_7 h_3 \succ_7 h_6 \succ_7 h_7 \succ_7 \ldots$
$\succ_8$: $h_9 \succ_8 h_5 \succ_8 h_1 \succ_8 h_8 \succ_8 \ldots$
$\succ_9$: $h_7 \succ_9 h_4 \succ_9 h_6 \succ_9 h_9 \succ_9 \ldots$

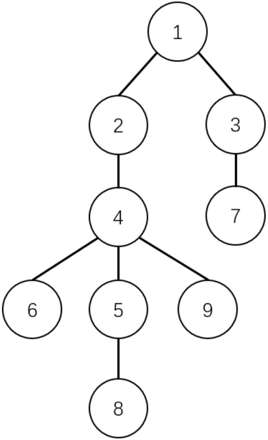
$order = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$



Compare LS with SWC

- LS is applicable in all networks, while SWC can only be applied in trees.
- In principle, each agent can exchange with all the others in LS, while in SWC only her subtree.
- In experiments, SWC is not different from SWN.

Compare LS with SWC



$$\succ_1: h_2 \succ_1 h_4 \succ_1 h_3 \succ_1 h_1 \succ_1 \ldots$$
$$\succ_2: h_4 \succ_2 h_3 \succ_2 h_5 \succ_2 h_2 \succ_2 \ldots$$
$$\succ_3: h_8 \succ_3 h_4 \succ_3 h_1 \succ_3 h_3 \succ_3 \ldots$$
$$\succ_4: h_1 \succ_4 h_5 \succ_4 h_3 \succ_4 h_4 \succ_4 \ldots$$
$$\succ_5: h_2 \succ_5 h_6 \succ_5 h_9 \succ_5 h_5 \succ_5 \ldots$$
$$\succ_6: h_4 \succ_6 h_1 \succ_6 h_8 \succ_6 h_6 \succ_6 \ldots$$
$$\succ_7: h_8 \succ_7 h_3 \succ_7 h_6 \succ_7 h_7 \succ_7 \ldots$$
$$\succ_8: h_9 \succ_8 h_5 \succ_8 h_1 \succ_8 h_8 \succ_8 \ldots$$
$$\succ_9: h_7 \succ_9 h_4 \succ_9 h_5 \succ_9 h_9 \succ_9 \ldots$$

$$order = \{1, 2, 3, 7, 4, 6, 5, 9, 8\}$$

Leave and Share

Compare LS with SWC

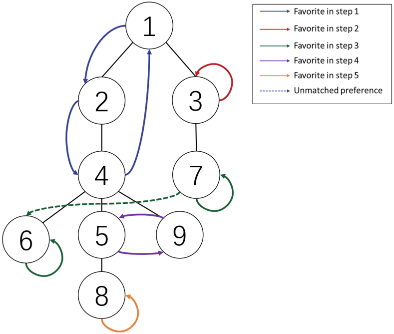


Figure: LS

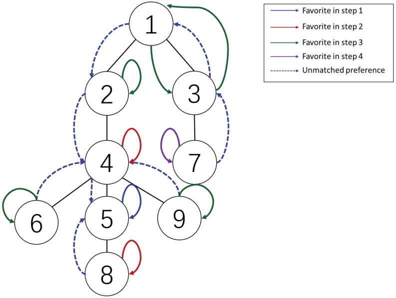


Figure: SWC

Open Questions

- How to further relax the restrictions of Swap with Neighbors?
- How to define optimal/stable in the network setting? What is a blocking pair?

Outline

Outline

1. Mechanism Design Review
2. Auctions
   - Information Diffusion Mechanism
   - Generalised IDM
   - Distance-Based Mechanism
3. Cooperative Games
   - The Shapley Value
   - Invitation Incentive Mechanisms
4. Matching
   - Top Trading Cycle
   - Leave and Share
5. Limited Reward Sharing
   - The Literature
   - Information Propagation

Invite More People to Share a Limited Reward

- Miners of bitcoin
  - Babaioff, M., Dobzinski, S., Oren, S., Zohar, A. *On Bitcoin and Red Balloons*. EC 2012.
- Lotteries
  - J. Chen and B. Li. *Maximal Information Propagation via Lotteries*. WINE 2021.
- All share
  - H. Shi, Y. Zhang, Z. Si, L. Wang, D. Zhao: *Maximal Information Propagation with Budgets*. ECAI 2020.

Miners of Bitcoin

The setting:

- Each miner has equal probability to win the reward, which is $1/n$
- Not all miners are in the game initially
- Duplication does NOT increase the chance of winning
- The network is a d-tree

The goal:

- incentivize miners to invite others
- prevent Sybil-attack

Ref: Babaioff, M., Dobzinski, S., Oren, S., Zohar, A. *On Bitcoin and Red Balloons*. EC 2012.

Lotteries

The setting:

- Each player has equal probability to win the reward, which is $1/n$
- Not all players are in the game initially
- Duplication does increase the chance of winning
- The network is a d-tree

The goal:

- incentivize players to invite others
- prevent Sybil-attack

Ref: J. Chen and B. Li. *Maximal Information Propagation via Lotteries*. WINE 2021.

Solution for Lotteries

Theorem

For $f \geq d \geq 3$, all agents fully propagating and withholding x_{min} is a Nash Equilibrium.

Suppose that each tree contains f nodes. When the root node withholds $2x_{min}$ instead of x_{min}, her tree will decrease to $\frac{f}{d}$ nodes.

1. when $f = d = 3$,
   - When withholding x_{min}, $u_1 = x_{min} \cdot \frac{f}{2f+f} = \frac{1}{3}x_{min}$.
   - When withholding $2x_{min}$, $u_2 = 2x_{min} \cdot \frac{f/3}{2f+f/3} = \frac{2}{7}x_{min}$.
   - $u_1 > u_2$, withholding x_{min} is better.
2. however, when $f = 3, d = 2$,
   - When withholding x_{min}, $u_1 = x_{min} \cdot \frac{f}{2f+f} = \frac{1}{3}x_{min}$.
   - When withholding $2x_{min}$, $u_2 = 2x_{min} \cdot \frac{f/2}{2f+f/2} = \frac{2}{5}x_{min}$.
   - $u_1 < u_2$, now withholding $2x_{min}$ is better.

All Share: No Special Winners

The setting:

- A fixed amount (from the sponsor) to be shared among all players
- Not all players are in the game initially
- Duplication is not considered
- No restriction on the network

The goal:

- incentivize players to invite others
- don't prevent Sybil-attack

Ref: H. Shi, Y. Zhang, Z. Si, L. Wang, D. Zhao: *Maximal Information Propagation with Budgets*. ECAI 2020.

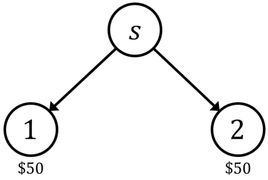
Outline

How to incentivize people to invite more to share a limited reward?

Suppose the sponsor has \$100 to be shared. Initially, agent 1 and agent 2 both get \$50.

Question

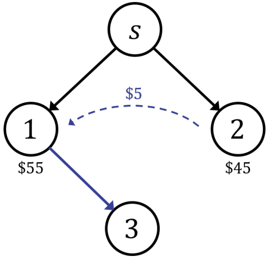
Question: How to incentivize agents 1 and 2 to invite more participants?



How to incentivize people to invite more to share a limited reward?

If agent 1 invites agent 3, how to distribute the budget \$100?

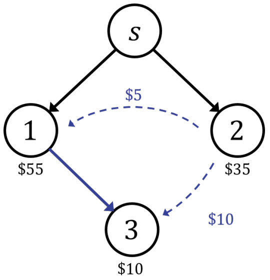
- Solution: give agent 1 (the inviter) a reward (\$5 for instance) from agent 2.



How to incentivize people to invite more to share a limited reward?

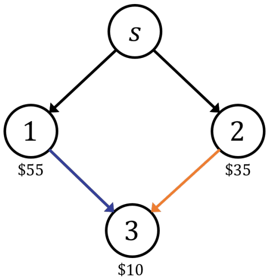
However, agent 3 will not join the game unless she is rewarded. Therefore, we should give agent 3 a reward ($10 for instance) from agent 2. By doing so:

- Agent 1 is incentivized to invite friends.
- Agent 3 is incentivized to join the game.



How to incentivize people to invite more to share a limited reward?

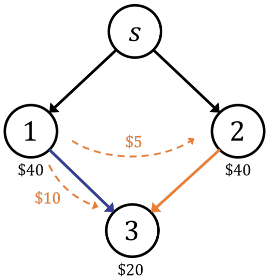
What if agent 2 also invites agent 3?



How to incentivize people to invite more to share a limited reward?

We use the same rewarding method, give the reward to agents 2 and 3 from agent 1.

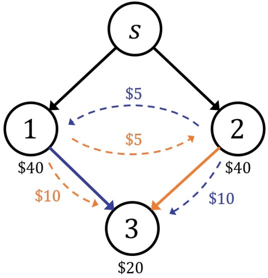
- Give \$5 to agent 2 as reward of inviting agent 3.
- Give \$10 to agent 3 to incentivize her to join.



How to incentivize people to invite more to share a limited reward?

For agent 1 and agent 2:

- If none of them invites, they both get $50.
- If one of them invites, then she get $55, another get $35.
- If both of them invite, they both get $40.



How to incentivize people to invite more to share a limited reward?

For agent 1 and agent 2:

- If none of them invites, they both get $50.
- If one of them invites, then she get $55, another get $35.
- If both of them invite, they both get $40.

Agent 1Agent 2	**Invite**	**NOT Invite**
Invite	40, 40	55, 35
NOT Invite	35, 55	50, 50

Conclusion

Invite agent 3 is dominant strategy for both agent 1 and 2 (peer pressure).

Outline