# Mechanism Design for Double Auctions with Temporal Constraints

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## Contributions

We introduce an extended double auction model where market clearing is restricted by temporal constraints. In this model, we propose a Vickrey-Clarke-Groves (VCG) mechanism based on bipartite matching.
The key contributions are

- an **efficient & monotonic allocation** based on maximum-weighted matching,
- a **faster algorithm** for computing VCG payment.

## The Model

Consider a double auction market where

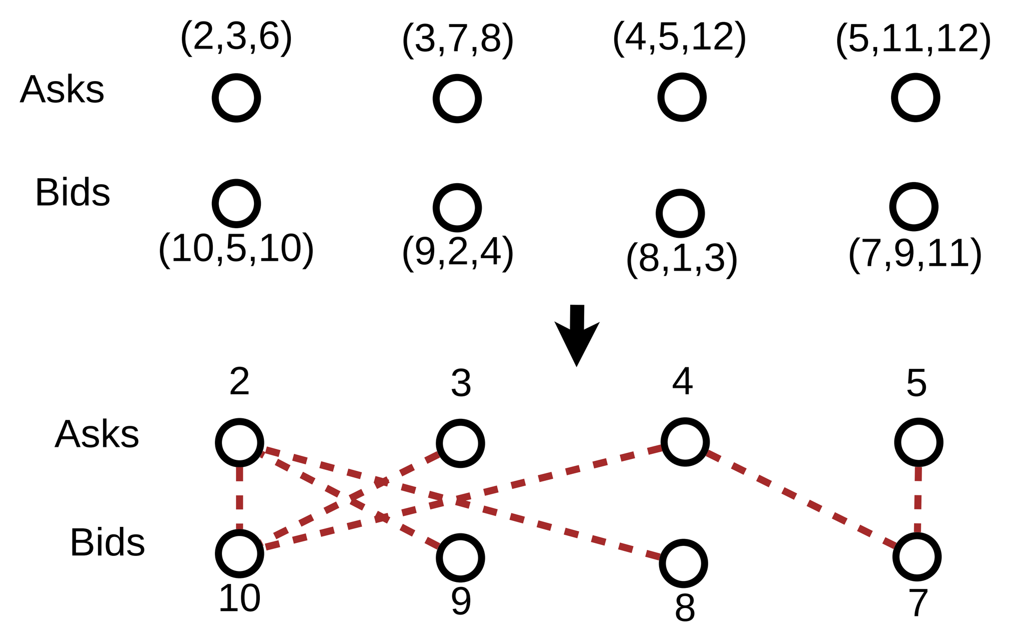
- **multiple** sellers and **multiple** buyers trade one commodity simultaneously,
- each seller/buyer supplies/demands a **single unit** of the commodity.

The **type** of trader (seller or buyer) $i$ is $\theta_i = (\mathbf{v_i}, \mathbf{s_i}, \mathbf{e_i})$, where

- $v_i$ is $i$'s valuation of a single unit of the commodity,
- $s_i$ and $e_i$ are the starting point and the ending point of the **time constraint** $[s_i, e_i]$.

We focus on **direct-revelation** mechanisms where traders are required to directly report their type to the auctioneer. We call a report from a seller **ask**, and a report from a buyer **bid**. An ask $\theta_i = (v_i, s_i, e_i)$ and a bid $\theta_i = (v_j, s_j, e_j)$ are **matchable** (i.e. item exchanging can happen between $i$ and $j$) iff $v_i \leq v_j$ and $[\mathbf{s_i}, \mathbf{e_i}] \cap [\mathbf{s_j}, \mathbf{e_j}] \neq \varnothing$.

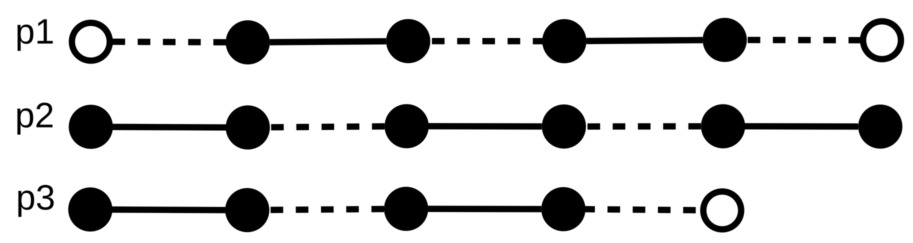
## Graphical Representation




The above graph shows an example of mapping reports into a bipartite graph. A **matching** $M$ in graph is a set of pair-wise non-adjacent edges. Given a matching $M$, an $M$**-alternating path** is a path in which the edges belong alternatively to $M$ and not to $M$.

- $M$**-augmenting path**: $M$-alternating path whose endpoints are free (unmatched).
- $M$**-abridging path**: $M$-alternating path whose first and last edge are in $M$.
- $M$**-replacement path**: $M$-alternating path with one endpoint in $M$ and the other is NOT in $M$.

The following gives an example for each of the above paths (lines are edges in the matching).




## Our Mechanism

The VCG mechanism is an **efficient** and **truthful** mechanism consisting of

1. an efficient **allocation policy**, i.e. it maximises the sum of the valuation of the traders who have goods in the end.
2. a **payment policy**, which is independent of the trader's valuation.
   - **Clarke pivot payment**, the classical VCG payment, charges each trader the harm he causes to other traders.

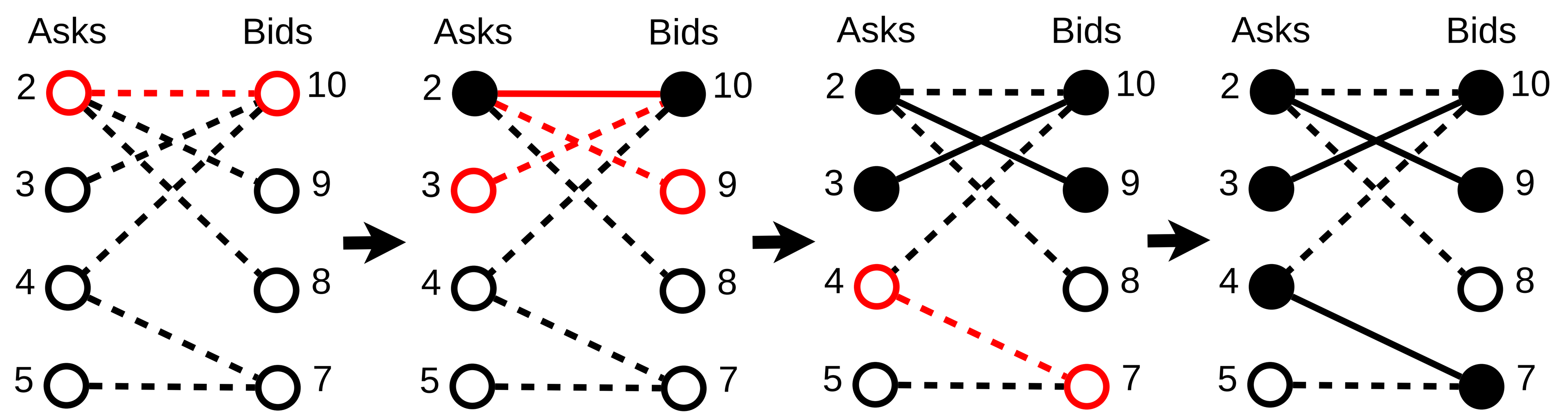
Based on the bipartite graph representation, our mechanism consists of

- an efficient & monotonic **maximum-weighted bipartite matching allocation**,
- an alternating path based **min-max payment**.

## Maximum-weighted Bipartite Matching Allocation

Constructs a maximum-weighted matching by

- beginning with the empty matching,
- **repeatedly performing augmentations** using **augmenting paths** of **maximum weight increase** until there is no more augmenting path with positive weight increase.



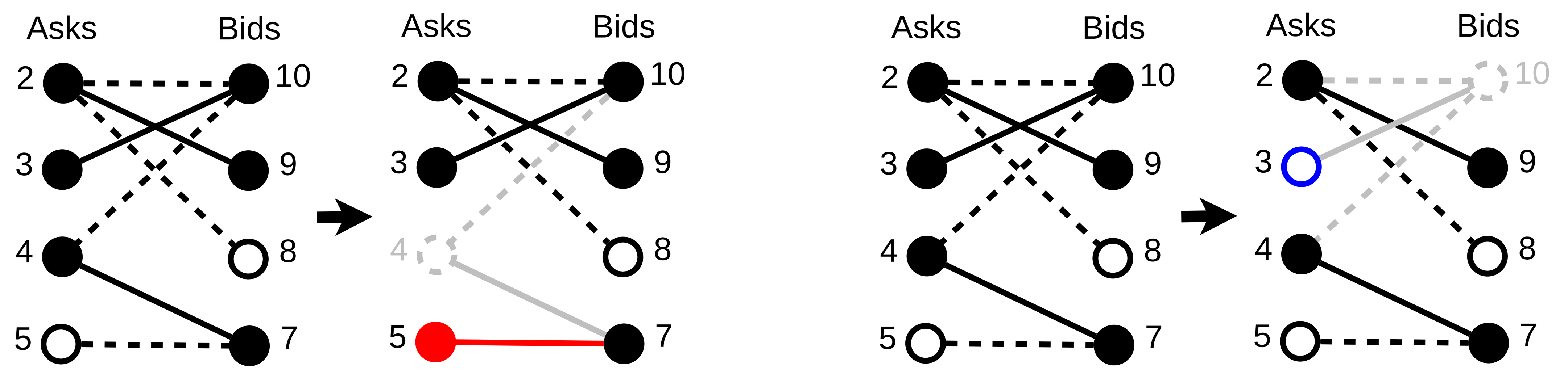

## Min-Max Payment

For each matched ask/bid, looking for **the best substitution** and **the lowest loss** if the ask/bid was not participated. We get this from the following abridging and replacement paths start from the ask/bid.

- **replacement paths** give all ways to remove the ask/bid by giving a substitution,
- **abridging paths** give all ways to remove the ask/bid by unmatching another bid/ask.

The payment is

- for an ask: the **minimum** valuation of all the substitutions and all possible bids to loss,
- for a bid: the **maximum** valuation of all the substitutions and all possible asks to loss.




## Properties of Our Mechanism

From the allocation:

- **Efficient**, i.e. maximising social welfare
- **Monotonic**, i.e. for any matched trader, if he reported a better ask/bid, he should be matched
- Complexity, can be implemented in $O(n^3)$, where $n$ is the number of traders

From the payment:

- **Truthful**, giving the same payment as Clarke pivot payment
- **Individual Rational**, i.e. no payment for unmatched traders
- Complexity
  - does **NOT** need to rerun the allocation for each matched ask/bid,
  - can be implemented $O(n)$ times **faster** than Clarke pivot payment.

## References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