

Multi-unit Double Auction under Group Buying

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Group Buying - Collective Buying



GROUPON
Collective Buying Power


livingsocial®

Group Buying Example

GROUPON
Collective Buying Power

Visit More Cities ▾ Get Daily Groupon Alerts ▾ Refer Friends, Get \$10 Contact Us

Daily Deals on the Best in
Houston

Total dollars saved
\$57,374,984

Total Groupons bought
1,144,019

Today's Deal Recent Deals How Groupon Works Discussion

Facebook Connect Sign in

Share This Deal: Facebook it! Tweet! Quick! Email a friend!

Today's Deal: \$30 for Nine Boot Camp Sessions, Gym Access, and Body Fat Analysis During One Month at Body3 Fitness Center (\$205 Value)

\$30 **Buy!**

Value	Discount	You Save
\$205	85%	\$175

Buy it for a friend!

Time Left To Buy
15 hours
5 minutes
42 seconds

42 bought

The deal is on!
Tipped at 8:37AM with 30 bought

The Fine Print
Expires 05/30/2010
Limit 1 per person. Not valid with other offers. Must call to schedule sessions. No cash back or credit.
[Read the Deal FAQ](#) for the basics.

Highlights

- Choose from two locations for the boot camp
- Daily classes with morning sessions available
- Full gym access including group classes

Discuss the Deal
 i was forwarded to voice mail. i hope i can... more
[Join the discussion!](#)
4 comments

Resolution Motivation
Reach your goals for 2010 with this week's special deals. Purchase any two separate deals before midnight on January 3rd and get \$10 in Groupon Bucks.
[Learn More](#)

Today's Side Deal
\$20 for One Month of Complete Pool Cleaning or Pool Repair from Mirah Pools (Up to \$140 Value)

Features of Existing Group Buying Platforms

Features

- one day one deal with big discount
- sellers are filtered
- a minimum number of purchases to make a deal on
- email, social networks (e.g. facebook)
- too much money flows to the company (50%)

Limitations of Existing Group Buying Platforms

Limitations

- price is predetermined and a deal can fail
- sellers' participation is limited
- buyers can't express their interest

We Want More...

What we want?

- 1 allow **sellers** to **compete** for a deal
- 2 a **richer valuation expression** for both buyers and sellers
- 3 **more transactions** (more flexible price)
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Mechanism Design

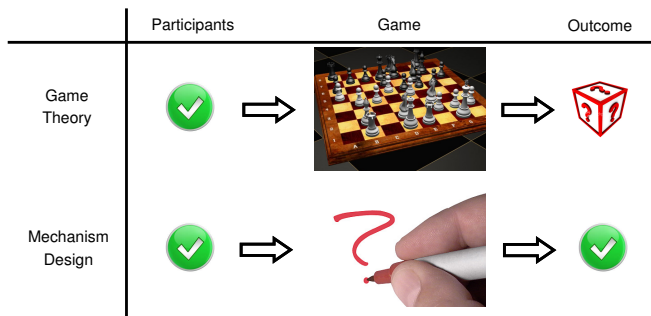
Outline

- 1 Background
 - Mechanism Design
 - Desiderata of Mechanism Design
- 2 Multi-unit Double Auction under Group Buying
 - The Goal
 - The Model
 - Theoretical Results
- 3 Conclusion

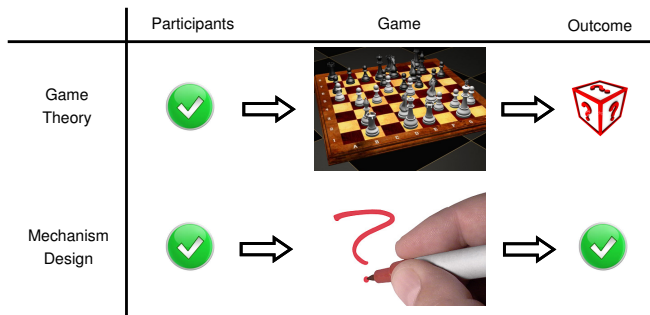
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Mechanism Design (Reverse Game Theory)



Mechanism Design (Reverse Game Theory)



Mechanism design answers...

How to **design** a mechanism which leads to a **desired outcome**?

Properties of the Outcomes

- IC incentive compatibility (truthfulness)
- Eff. social-welfare maximisation (efficiency)
- IR individual rationality (no agent worse off)
- (W)BB (weak) budget balance (zero profit for the market owner)

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The Goal

is to satisfy

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The Goal

Setting Related

- 1 allow **sellers** to **compete** for a deal
- 2 a **richer valuation expression** for both buyers and sellers

Design Related

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The Goal

Setting Related

- 1 allow **sellers** to **compete** for a deal
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- 2 a **richer valuation expression** for both buyers and sellers
 - monotonic valuation with group buying discount

Design Related

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 - maximising trading size
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Basic Setting

A multi-unit double auction:

- 1 multiple sellers
- 2 multiple buyers
- 3 one commodity
- 4 each trader supplies/demands multiple units

Richer Valuation Expression

Trader i has valuation function $v_i : \mathbb{Z} \rightarrow \mathbb{R}$.

- Seller:

- unlimited supply
- **monotonic**: $v_i(k) \leq v_i(k+1)$
- **group buying discount**: $\frac{v_i(k)}{k} \geq \frac{v_i(k+1)}{k+1}$

- Buyer:

- demands c_i units
- $v_i(k) = v_i(c_i) > 0$ for all $k \geq c_i$, otherwise $v_i(k) = 0$

Setting Fixed!

Setting Related

- ① allow sellers to compete for a deal
 - multiple sellers
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Design Related

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The Design Task

design mechanisms s.t.

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- 2 (weak) budget balance
- 3 truthfulness
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Existence of IC, IR, BB Auctions

Theorem

There exists a (weakly) budget balanced, individually rational, and truthful multi-unit double auction.

Existence of IC, IR, BB Auctions

Existence Examples

- **do nothing!** i.e. no transaction, no payments.
- **fixed-price auctions**, i.e. price doesn't depend on traders.

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Question

Can we have something other than these?

Impossibility I

There is **NO** (weakly) budget balanced, individually rational and truthful multi-unit double auction,
given that

- *both the trading size and the payment are neither seller-independent nor buyer-independent.*

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Definition

We say a parameter of a double auction is **seller-independent** (**buyer-independent**) if the value of the parameter does not depend on sellers' (buyers') type reports.

Impossibility I

There is **NO** (weakly) budget balanced, individually rational and truthful multi-unit double auction,
given that

- both *the trading size* and *the payment* are neither *seller-independent* nor *buyer-independent*.

Why?

buyers want to form a bigger group while sellers might not!

- buyers with **larger** group will lower their payments
- a seller's profit might **not** maximised when selling **more**

Partially Truthful Auctions

Theorem

*There exist (weakly) budget balanced, individually rational, and **one-sided truthful** multi-unit double auctions, given that both the trading size and the payment are neither seller-independent nor buyer-independent.*

Partially Truthful Auctions

Second Price Auction

Given type profile report $v = (v^B, v^S)$, assume that $v_1^B(1) \geq v_2^B(1) \geq \dots \geq v_m^B(1)$.

- ① Let $w(k) = \min \arg \min_i v_i^S(k)$ and $p(k) = \min_{i \neq w(k)} \frac{v_i^S(k)}{k}$ or ∞ if there is only one seller.
- ② Let $k^* = \max\{k \mid v_k^B(1) \geq p(k)\}$.
- ③ The first k^* buyers, i.e. buyers of valuation $v_1^B, v_2^B, \dots, v_{k^*}^B$, receive one unit of the commodity each and each of them pays $p(k^*)$.
- ④ Seller $w(k^*)$ sells k^* units of the commodity and receives payment $p(k^*) \cdot k^*$.
- ⑤ The remaining traders lose without payment.

Not Done Yet!

We faced impossibility!

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Impossibility II

- There is no (weakly) budget balanced, individually rational, truthful multi-unit double auction that can **guarantee trading size**.

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truthfulness, individual rationality

Future Work

- ① limited supply case, e.g. social welfare, utility calculation
 - For instance, a seller supplies two units with unit prices $p_1 > p_2$ for selling one and two units respectively. If we end up with one unit left for the seller, we might consider that the seller has a valuation of p_1 for this unsold unit.
- ② online multi-unit double auction, i.e. the advertising effect
 - How many buyers will return?
 - Will they tell the product to others?

Q & A

Thank you for your attention!