Multi-unit Double Auction under Group Buying

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Group Buying - Collective Buying

Group Buying Example

Features of Existing Group Buying Platforms

Features

- one day one deal with big discount
- sellers are filtered
- a minimum number of purchases to make a deal on
- email, social networks (e.g. facebook)
- too much money flows to the company (50%)

Limitations of Existing Group Buying Platforms

Limitations

- price is predetermined and a deal can fail
- sellers' participation is limited
- buyers can't express their interest

We Want More...

What we want?

1. allow sellers to compete for a deal
2. a richer valuation expression for both buyers and sellers
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Mechanism Design

Outline

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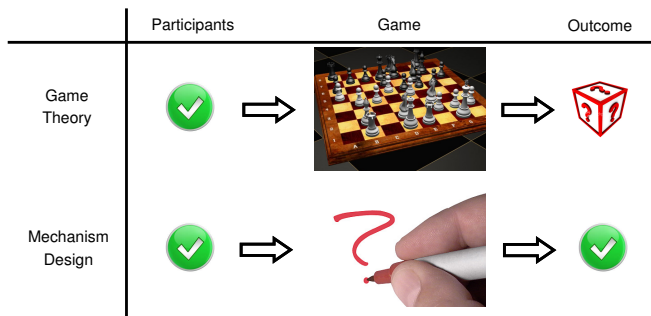
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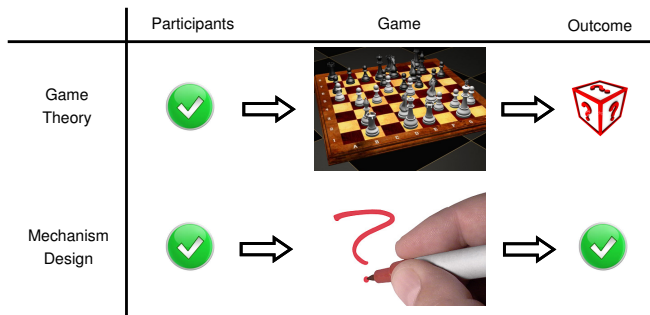
Mechanism design answers...

How to design a mechanism which leads to a desired outcome?

Desiderata of Mechanism Design

Properties of the Outcomes

IC   incentive compatibility (truthfulness)

Eff.   social-welfare maximisation (efficiency)

IR   individual rationality (no agent worse off)

(W)BB   (weak) budget balance (zero profit for the market owner)

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is to satisfy

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Setting Related

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Design Related

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The Goal

Setting Related

1. allow sellers to compete for a deal
   - multiple sellers
2. a richer valuation expression for both buyers and sellers
   - monotonic valuation with group buying discount

Design Related

1. more transactions (more flexible price)
   - maximising trading size
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Basic Setting

A multi-unit double auction:

1. multiple sellers
2. multiple buyers
3. one commodity
4. each trader supplies/demands multiple units

Richer Valuation Expression

Trader i has valuation function $v_i : \mathbb{Z} \to \mathbb{R}$.

- Seller:
  - unlimited supply
  - monotonic: $v_i(k) \leq v_i(k+1)$
  - group buying discount: $\frac{v_i(k)}{k} \geq \frac{v_i(k+1)}{k+1}$
- Buyer:
  - demands c_i units
  - $v_i(k) = v_i(c_i) > 0$ for all $k \geq c_i$, otherwise $v_i(k) = 0$

The Model

Setting Fixed!

Setting Related

1. allow sellers to compete for a deal
   - multiple sellers
2. a richer valuation expression for both buyers and sellers
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Design Related

1. more transactions (more flexible price determination)
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The Design Task

design mechanisms s.t.

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2. (weak) budget balance
3. truthfulness
4. individual rationality

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Existence of IC, IR, BB Auctions

Theorem

There exists a (weakly) budget balanced, individually rational, and truthful multi-unit double auction.

Existence of IC, IR, BB Auctions

Existence Examples

- **do nothing!** i.e. no transaction, no payments.
- fixed-price auctions, i.e. price doesn't depend on traders.

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Question

Can we have something other than these?

Theoretical Results

Impossibility I

There is NO (weakly) budget balanced, individually rational and truthful multi-unit double auction,

given that

- *both the trading size and the payment are neither seller-independent nor buyer-independent.*

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Definition

We say a parameter of a double auction is seller-independent (buyer-independent) if the value of the parameter does not depend on sellers' (buyers') type reports.

Impossibility I

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given that

- *both the trading size and the payment are neither seller-independent nor buyer-independent.*

Why?

buyers want to form a bigger group while sellers might not!

- buyers with larger group will lower their payments
- a seller's profit might not maximised when selling more

Partially Truthful Auctions

Theorem

There exist (weakly) budget balanced, individually rational, and *one-sided truthful* *multi-unit double auctions,* `given that both the trading size and the payment are neither seller-independent nor buyer-independent.`

Partially Truthful Auctions

Second Price Auction

Given type profile report $v = (v^B, v^S)$, assume that
$v_1^B(1) \geq v_2^B(1) \geq \cdots \geq v_m^B(1)$.

1. Let $w(k) = \min \operatorname{argmin}_i v_i^S(k)$ and $p(k) = \min_{i \neq w(k)} \frac{v_i^S(k)}{k}$ or ∞ if there is only one seller.

2. Let $k^* = \max\{k | v_k^B(1) \geq p(k)\}$.

3. The first k^* buyers, i.e. buyers of valuation $v_1^B, v_2^B, \cdots, v_{k^*}^B$, receive one unit of the commodity each and each of them pays $p(k^*)$.

4. Seller $w(k^*)$ sells k^* units of the commodity and receives payment $p(k^*) \cdot k^*$.

5. The remaining traders lose without payment.

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Not Done Yet!

We faced impossibility!

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Impossibility II

- There is no (weakly) budget balanced, individually rational, truthful multi-unit double auction that can guarantee trading size.

Outline

Summary

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2. a richer valuation expression for both buyers and sellers
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truthfulness, individual rationality

Future Work

1. limited supply case, e.g. social welfare, utility calculation
   - For instance, a seller supplies two units with unit prices $p_1 > p_2$ for selling one and two units respectively. If we end up with one unit left for the seller, we might consider that the seller has a valuation of p_1 for this unsold unit.

2. online multi-unit double auction, i.e. the advertising effect
   - How many buyers will return?
   - Will they tell the product to others?

Q & A

Thank you for your attention!