Background	Existing Matching	Maximal Matching	Experiments 0000	Conclusion

# Maximal Matching for Double Auction

### Dongmo Zhang<sup>1</sup> Dengji Zhao<sup>1,2</sup> Md Khan<sup>1</sup> Laurent Perrussel<sup>2</sup>

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<sup>2</sup>IRIT, University of Toulouse, France

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Background	Existing Matching	Maximal Matching	Experiments 0000	Conclusion

# Stock Exchanges

		X	Shar	e in	Au	stra	lia	+ and per
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	0.09		H POYNTON	06,00	0.94	0.94	38T	INSURAL
	0.04		HALL GOLD	0.23	0,25	0.23	0	TRACT
	0.26			0.29	10.31	0.295	LIT	
	0.39		HAMPTON	0.045	0.06	0.045		Name -
	048	U	HANCOCK GR	- 60	.62	1.60	5T	
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	.075	TUT	HARDIE J. HARDMAN	-5 28/		3.30	4417.	RS.
	0.34		HARGRAVES	0.96	O.Pr	0.063	48	TO ISP
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en u	004		HAWKER	8.65	-0n		97T	ALL ORD
	1=69	0	HAZEL TON	2.21		8.65	44T	TRONG

Background	Existing Matching	Maximal Matching	Experiments	Conclusion

# How to Choose Stock Exchanges

### Question

Which stock market will you choose?

Background	Existing Matching	Maximal Matching	Experiments 0000	Conclusion

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# How to Choose Stock Exchanges

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Which stock market will you choose?

The one...

- you can earn more money
- has higher chance to get traded
- most other people go

Background	Existing Matching	Maximal Matching	Experiments	Conclusion

# How to Choose Stock Exchanges

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How do you know?

Background	Existing Matching	Maximal Matching	Experiments 0000	Conclusion
How to (	Choose Stock	Exchanges		

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How do you know?

Market Liquidity

Background	Existing Matching	Maximal Matching	Experiments 0000	Conclusion
What is M	larket Liquidit	y?		

- number of transactions
- trade volume (buy/sell-volume)
  - the sum of the price of transacted orders

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Background	Existing Matching	Maximal Matching	Experiments 0000	Conclusion
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Can a stock market owner improve market liquidity to get more traders and more profit?

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Background	Existing Matching	Maximal Matching	Experiments 0000	Conclusion
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# What is Market Liquidity?

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Can a stock market owner improve market liquidity to get more traders and more profit?

# **Double Auction**

Background	Existing Matching	Maximal Matching	Experiments	Conclusion
Outline				

- Background
  - Double Auction
- 2 Existing Matching
  - Equilibrium Matching
- 3 Maximal Matching
  - The Algorithm
  - Properties of Maximal Matching
- 4 Experiments
  - Settings
  - Results



Background	Existing Matching	Maximal Matching	Experiments 0000	Conclusion
Outline				

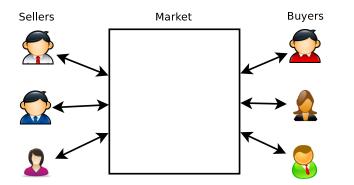


- 2 Existing Matching
- 3 Maximal Matching
- 4 Experiments
- 5 Conclusion

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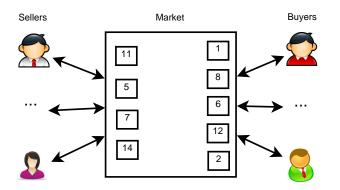
Background ●ooo	Existing Matching	Maximal Matching	Experiments	Conclusion
Double Auction				
Model				

- Three roles: seller, buyer, and market maker.
- One commodity, e.g. google's stocks.



Background ●ooo	Existing Matching	Maximal Matching	Experiments	Conclusion
Double Auction				
Model				

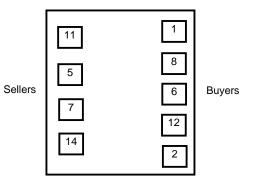
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Background o●oo	Existing Matching	Maximal Matching	Experiments	Conclusion
Double Auction				
Exchang	ging Rules			

### For market maker:

- which sell offer to be matched with which buy offer?
- what is the price for each match?



Market

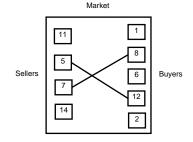
Background oo●o	Existing Matching	Maximal Matching	Experiments	Conclusion
Double Auction				
Definition	S			

- Ask: offer (price) submitted by a seller, the minimum price willing to sell
- **Bid**: offer (price) submitted by a buyer, the maximum price willing to buy
- Matching: a set of pairs of ask and bid, where in each pair
  - bid's price  $\geq$  ask's price
  - no bid or ask belongs to more than one pair
- Market Liquidity:
  - number of transactions (matching size)
  - trade volume (buy/sell-volume)
    - buy-volume: the sum of the price of transacted bids
    - sell-volume: the sum of the price of transacted asks
- Auctioner's Profit: The price difference between matched bids and asks

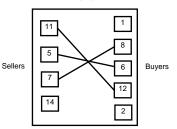
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Background 000●	Existing Matching	Maximal Matching	Experiments	Conclusion
Double Auction				
Matching	g Examples			

- bid's price  $\geq$  ask's price
- no bid or ask belongs to more than one pair



Market



$$Profit = (12+8)-(5+7) = 8$$

Profit = (12+8+6)-(5+7+11) = 3

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Background	Existing Matching	Maximal Matching	Experiments	Conclusion
Outline				



- Existing Matching
   Equilibrium Matching
- 3 Maximal Matching
- 4 Experiments

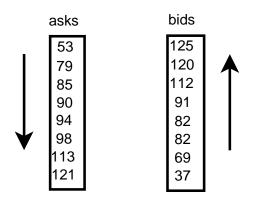
## 5 Conclusion

Background	Existing Matching	Maximal Matching	Experiments	Conclusion
Equilibrium Matching				
Main Idea				

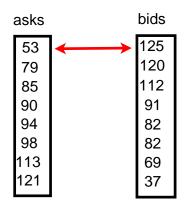
- Sort all asks (bids) in ascending (descending) order w.r.t. their price.
- ❷ Based on this sort order, starting at the top, add each ask-bid pair to the result matching, if ask's price ≤ bid's price.

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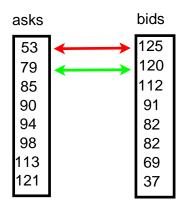
Background	Existing Matching	Maximal Matching	Experiments	Conclusion
Equilibrium Matching				
Main Idea				



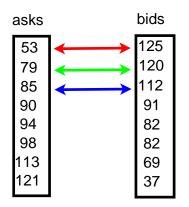
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Equilibrium Matching				
Main Idea				



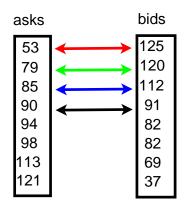
Background	Existing Matching ●○	Maximal Matching	Experiments	Conclusion
Equilibrium Matching				
Main Idea				



Background	Existing Matching ●○	Maximal Matching	Experiments	Conclusion
Equilibrium Matching				
Main Idea				

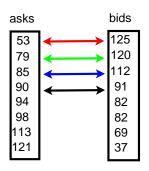


Background	Existing Matching ●○	Maximal Matching	Experiments	Conclusion
Equilibrium Matching				
Main Idea				



Background	Existing Matcl	ning	Maximal Matching	Experiments 0000	Conclusion
Equilibrium Match	ing				
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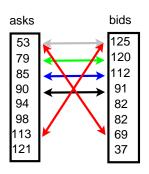
## Properties of Equilibrium Matching



- profit maximizing (141)
- e market liquidity can be improved
  - transactions: 4
  - 2 buy/sell-volume: 448/307

Background	Existing Matching ○●	Maximal Matching	Experiments	Conclusion
Equilibrium Match	ing			

## Properties of Equilibrium Matching



- profit maximizing (141)/97
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  - transactions: 4
  - 2 buy/sell-volume: 448/307

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Background	Existing Matching	Maximal Matching	Experiments 0000	Conclusion
Outline				



- 2 Existing Matching
- 3 Maximal Matching
  - The Algorithm
  - Properties of Maximal Matching

## Experiments

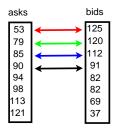
# 5 Conclusion

Background	Existing Matching	Maximal Matching	Experiments 0000	Conclusion
The Algorithm				
What We	Want?			

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- Maximizing market liquidity
- Keeping as much profit as we can

Background	Existing Matching	Maximal Matching	Experiments	Conclusion
The Algorithm				
General	Idea			



Equilibrium Matching

- Starting from equilibrium matching
- Matching unmatched shouts (asks and bids) as many as we can
- Oecreasing profit as less as we can

Background	Existing Matching	Maximal Matching	Experiments	Conclusion
The Algorithm				

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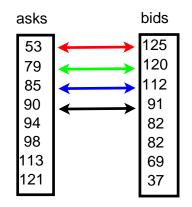
## Looking Extra Matchable Shouts

### Question

How to find extra matchable shouts?

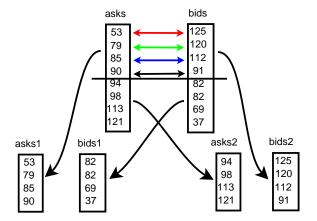
Background	Existing Matching	Maximal Matching	Experiments 0000	Conclusion
The Algorithm				
Looking	Extra Matcha	ble Shouts		

### Equilibrium Matching



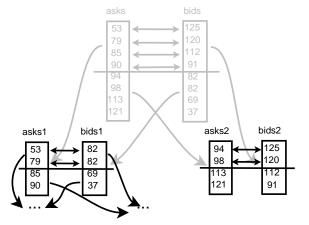
Background	Existing Matching	Maximal Matching	Experiments	Conclusion
The Algorithm				
Looking	Extra Matcha	ble Shouts		

### **Decompose Matching**

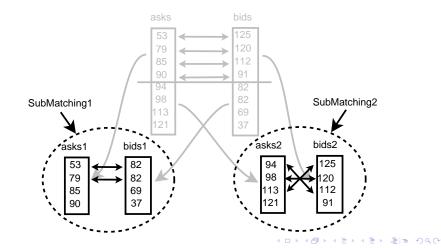


Background	Existing Matching	Maximal Matching	Experiments	Conclusion
The Algorithm				
Looking	Extra Matcha	ble Shouts		

## Maximal Matching in Sub-matchings

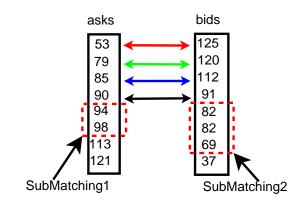


Background	Existing Matching	Maximal Matching	Experiments	Conclusion
The Algorithm				
Looking	Extra Matcha	ble Shouts		
	Maximal M	atching in Sub-ma	atchings	



Background	Existing Matching	Maximal Matching	Experiments	Conclusion
The Algorithm				
Looking	Extra Matcha	ble Shouts		

Extra Matchable Shouts



Backg	round

Existing Matching

Maximal Matching

Experiments

Conclusion

The Algorithm

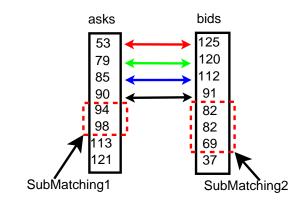
## Decreasing Profit as Less as We Can

### Question

How to keep profit as much as we can?

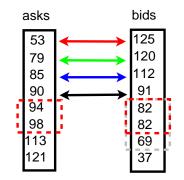
Background	Existing Matching	Maximal Matching	Experiments	Conclusion
The Algorithm				
Decreasing Profit as Less as We Can				

Extra Matchable Shouts



Background	Existing Matching	Maximal Matching	Experiments 0000	Conclusion
The Algorithm				
Decreasin	n Profit as I	ess as We Ca	an	

Removing Bad Ones (Balancing)



Background	Existing Matching	Maximal Matching	Experiments	Conclusion
The Algorithm				

# **Cross-Match Matchable Shouts**

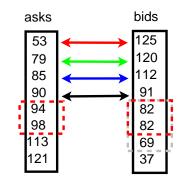
### Question

How to match extra matchable ones?

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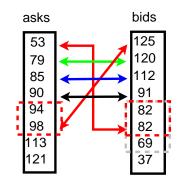
Background	Existing Matching	Maximal Matching	Experiments	Conclusion
The Algorithm				
Cross-Ma	atch Matchat	ole Shouts		

Final Extra Matchable Shouts



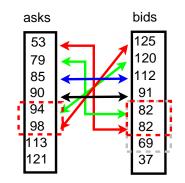
Background	Existing Matching	Maximal Matching	Experiments	Conclusion
The Algorithm				
Cross-M	latch Matchat	ble Shouts		

### **Cross-Matching**



Background	Existing Matching	Maximal Matching	Experiments	Conclusion
The Algorithm				
Cross-M	latch Matchat	ble Shouts		

### **Cross-Matching**



Background	Existing Matching	Maximal Matching	Experiments	Conclusion
The Algorithm				
The Algo	orithm			

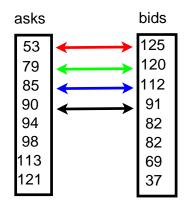
### Algorithm 3.1: MaximalMatching

- *Matching* ← EquilibriumMatching(*Asks*, *Bids*);
- 2 MatchedAsks ← all asks from Matching in ascending order;
- *MatchedBids* ← all bids from *Matching* in descending order;
- *MM*1 ← MaximalMatching(*MatchedAsks*, (*Bids* \ *MatchedBids*));
- $MM2 \leftarrow MaximalMatching((Asks \setminus MatchedAsks), MatchedBids);$

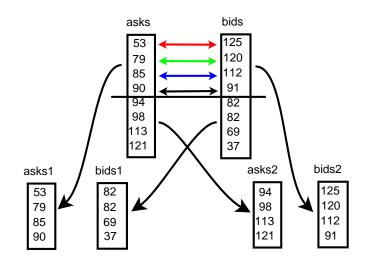
- *NumberOfExtraMatchableMatches* ← *Min*(|*MM*1|, |*MM*2|);
- 7 Cross-match extra matchable asks and bids;

Background	Existing Matching	Maximal Matching	Experiments	Conclusion
The Algorithm				
The Algo	orithm			

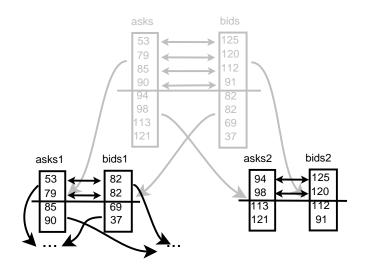
### Equilibrium Matching



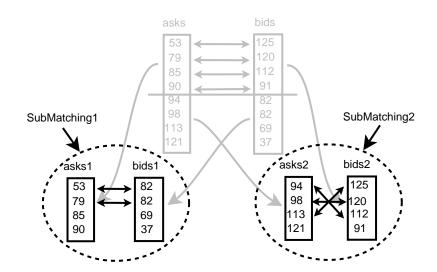
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The Alg	orithm			



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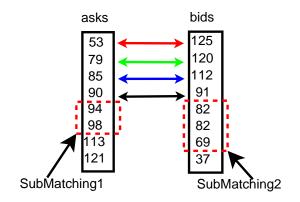


Background	Existing Matching	Maximal Matching	Experiments	Conclusion
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The Algo	orithm			



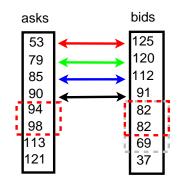
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Background	Existing Matching	Maximal Matching	Experiments	Conclusion
The Algorithm				
The Alg	orithm			

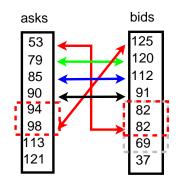


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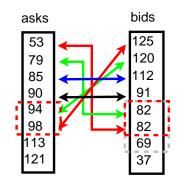
Background	Existing Matching	Maximal Matching	Experiments 0000	Conclusion
The Algorithm				
The Alg	orithm			



Background	Existing Matching	Maximal Matching	Experiments	Conclusion
The Algorithm				
The Alg	orithm			



Background	Existing Matching	Maximal Matching	Experiments	Conclusion
The Algorithm				
The Alg	orithm			



Background	Existing Matching	Maximal Matching	Experiments 0000	Conclusion
The Algorithm				
Complexity	y of Maximal I	Matching		

Worst case performance:

•  $O(\max(n_a, n_b) \log \max(n_a, n_b) + \min(n_a, n_b)^2)$ , where  $n_a$ ( $n_b$ ) is the number of asks (bids).

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Background	Existing Matching	Maximal Matching	Experiments	Conclusion
Properties of Maxin	nal Matching			
The Mai	n Result			

*Maximizing Trasactions*: Given a set of shouts, the size of maximal matching is maximal.

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Background	Existing Matching	Maximal Matching	Experiments	Conclusion
Properties of Maximal	Matching			
The Main	Result			

*Maximizing Trasactions*: Given a set of shouts, the size of maximal matching is maximal.

### Sketch proof.

- Induction:
  - assume the two sub-matchings are maximal
  - then the parent matching is also maximal

### 2 Base:

• no bid's price  $\geq$  any ask's price

Background	Existing Matching	Maximal Matching	Experiments	Conclusion
Properties of Maxin	nal Matching			
The Mai	n Result			

*Maximizing Trasactions*: Given a set of shouts, the size of maximal matching is maximal.

#### Theorem

*Maximizing Sell-Volume/Profit*: Given a set of shouts, maximal matching maximizes buy-volume and minimizes sell-volume. Auctioneer's profit is maximal among all matchings with the same size.

Background	Existing Matching	Maximal Matching	Experiments	Conclusion
Properties of Maxin	nal Matching			
The Mai	n Result			

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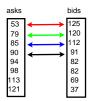
#### Proof.

Because of the descending (ascending) order of bids (asks), and maximal matching always matches the first n biggest (smallest) bids (asks), where n is the size of the matching.

	Conclusion
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#### Properties of Maximal Matching

# Equilibrium Matching vs Maximal Matching

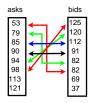


## Equilibrium Matching

- profit maximizing (141)
- market liquidity can be improved
  - transactions: 4
  - buy/sell-volume: 448/307

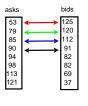
### **Maximal Matching**

- profit maximizing (conditional) (113)
- market liquidity maximizing
  - transactions: 6
  - 2 buy/sell-volume: 612/499



Background	Existing Matching	Maximal Matching ○○○○○○○○●○	Experiments 0000	Conclusion
Properties of Maximal M	Matching			
Maximal N	Matching is R	eally Nice?		

### From economic point of view:





## **Equilibrium Matching**

- either *incentive compatible*
- or efficient
- oprofit maximizing

### **Maximal Matching**

- not incentive compatible
- Inot efficient
- less profit

Backgr	ound	

Existing Matching

Maximal Matching

Experiments

Conclusion

Properties of Maximal Matching

# Empirical Findings in the Long Term

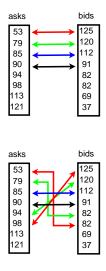
### Question

Can a stock market owner improve market liquidity to get more traders and more profit?

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Background	Existing Matching	Maximal Matching	Experiments 0000	Conclusion
Properties of Maximal M	atching			

# Empirical Findings in the Long Term



### **Equilibrium Matching**

- less traders
- less profit
- **Maximal Matching** 
  - more traders

e more profit

Background	Existing Matching	Maximal Matching	Experiments	Conclusion
Outline				



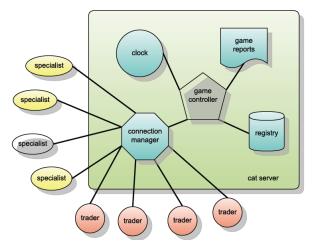
- 2 Existing Matching
- 3 Maximal Matching





Background	Existing Matching	Maximal Matching	Experiments ••••	Conclusion
Settings				
Test Platf	orm			

### Trading-Agent Competition Market Design (CAT)



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Background	Existing Matching	Maximal Matching	Experiments	Conclusion
Settings				
Test Sett	ings			

### Markets:

- EM: with equilibrium matching
- MM: with maximal matching

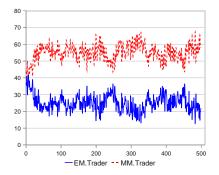
### Traders:

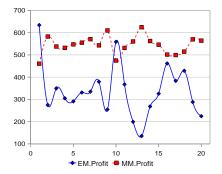
- 80 (40 sellers and 40 buyers) with profit seeking strategies
- they can only submit offers to sell or buy one goods
- not allowed to have more than one offer at the same time

### Others:

- 500 virtual days, 10 rounds in each day
- each trader only chooses one market to trade in each day
- each trader has a fixed number of goods, say 3, to trade in each day

Background	Existing Matching	Maximal Matching	Experiments	Conclusion
Results				
Experim	ental Results			





### Trader Distribution

### Auctioneer's Profit (avg/25ds)

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= 990

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Background	Existing Matching	Maximal Matching	Experiments	Conclusion
Results				
Our CAT	Specialist: <i>ja</i>	ickaroo		

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jackaroo achievements (leader: Dongmo Zhang, UWS):

- CAT 2007: 4th
- CAT 2008: 3rd
- CAT 2009: Champion
- CAT 2010: 2nd

Background	Existing Matching	Maximal Matching	Experiments	Conclusion
Outline				

# Background

- 2 Existing Matching
- 3 Maximal Matching

### 4 Experiments





Background	Existing Matching	Maximal Matching	Experiments 0000	Conclusion
Summary				

Matching for Double Auction

- Equilibrium Matching
- Maximal Matching
  - maximizes market liquidity
  - maximizes profit (conditional)
  - empirical findings
    - attracting traders
    - increasing profit

Further research directions:

- Online Double Auction, i.e. adding temporal information,
  - e.g. a sell offer would look like...
     "I want to sell a house only between Jan 2011 and March 2011."

Background	Existing Matching	Maximal Matching	Experiments 0000	Conclusion
Acknowle	edgments			

- **People**: Dongmo Zhang, Laurent Perrussel, Md Khan, *jackaroo* team, and anonymous reviewers.
- **Funding**: the Australian Research Council Discovery Project DP0988750.

Background	Existing Matching	Maximal Matching	Experiments	Conclusion
Acknow	ledgments			

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- **Funding**: the Australian Research Council Discovery Project DP0988750.

### Thank you for your attention!

# Outline







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# Complexities

Worst case performance:

- Maximal Matching:
  - $O(\max(n_a, n_b) \log \max(n_a, n_b) + \min(n_a, n_b)^2)$ , where  $n_a$ ( $n_b$ ) is the number of asks (bids).
- the Hopcroft-Karp algorithm:
  - O(|E|√n<sub>a</sub> + n<sub>b</sub>), where |E| ≥ (n<sub>em</sub>)<sup>2</sup>, n<sub>em</sub> is the size of equilibrium matching in our model.

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# Outline



- Online Mechanism Design
  - Motivation
  - Online Mechanism Design Examples

#### Motivation

# Why Online?

Mechanism Design has focused on static settings where

- no uncertainty,
- the participants are known and independent,
- (mostly) only one decision to make.

But many real environments are dynamic in the sense of that

- the number of participants is changing,
- the private information of participants is changing.

#### Examples

- stock exchanges.
- peer-to-peer file sharing (e.g. BitTorrent).
- allocating computational resources (e.g. CPU time) to jobs arriving over time.

#### Motivation

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- allocating computational resources (e.g. CPU time) to jobs arriving over time.

**Online Mechanism Design Examples** 

Example I (Dynamic Buyers)

**Online Vickrey Auction** 



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**Online Mechanism Design Examples** 

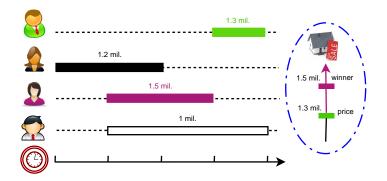
# Example I (Dynamic Buyers)

#### Vickrey Auction (second-price sealed-bid)



# Example I (Dynamic Buyers)

### **Online Vickrey Auction**



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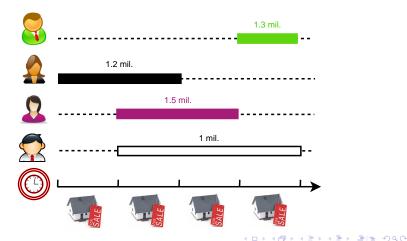
**Online Mechanism Design Examples** 

Example II (Dynamic Buyers)

#### Selling many identical houses (goods) in different time

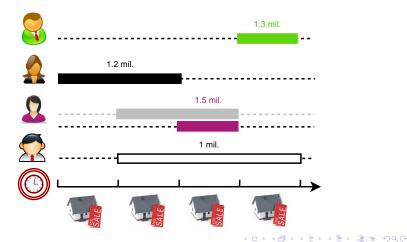
# Example II (Dynamic Buyers)

### Selling many identical houses (goods) in different time



# Example II (Dynamic Buyers)

### Selling many identical houses (goods) in different time



**Online Mechanism Design Examples** 

Example III (Dynamic Seller)

Ad Auction





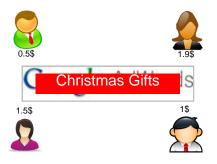
Complexities

Online Mechanism Design

**Online Mechanism Design Examples** 

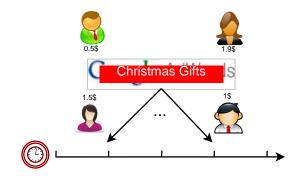
### Example III (Dynamic Seller)

#### Ad Auction: buyers bid for "Keyword"



### Example III (Dynamic Seller)

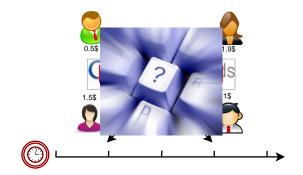
#### Ad Auction: dynamic arrival of "Keyword"



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### Example III (Dynamic Seller)

#### Ad Auction: how many "Keyword" will arrive?



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**Online Mechanism Design Examples** 

### Example IV (Dynamic Sellers and Buyers)

Exchanges: stock, currency, futures...

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**Online Mechanism Design Examples** 

### Example IV (Dynamic Sellers and Buyers)

#### Exchanges: stock, currency, futures...

#### **Double Auction**

**Online Mechanism Design Examples** 

### Example IV (Dynamic Sellers and Buyers)

Exchanges: stock, currency, futures...

#### **Online** Double Auction

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**Online Mechanism Design Examples** 

# Summary

- Dynamics from buyers
  - online Vickrey auction (one goods to sell)
  - many goods to sell in a fixed schedule
- Dynamics from sellers
  - Ad auctions
- Dynamics from both
  - online double auction (exchanges)

Complexities

Online Mechanism Design

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**Online Mechanism Design Examples** 



- dynamics provide new strategies for participants
- solutions of static mechanism design are insufficient

Online Mechanism Design Examples

# Static vs Online Mechanism Design

- (Static) Mechanism Design
  - well studied since 60s
  - got many nice results (e.g. Vickery auctions)
- Online Mechanism Design
  - just addressed (since 2000)
  - many real environments are dynamic, e.g. exchanges
  - new challenges (uncertainties of future)

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**Online Mechanism Design Examples** 

## What We Bring?

### Economists:

- incentive constraint
- Computer Scientists:
  - computational constraint