Selling Multiple Items via Social Networks

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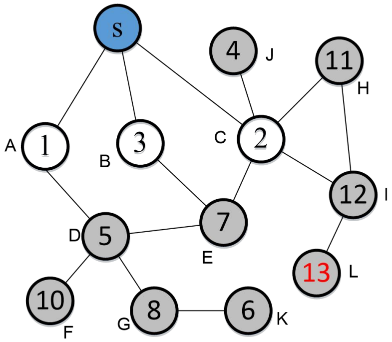
AAMAS'18

Starter: Promote a Sale in Social Networks



- The seller (blue node) sells one item and has only three connections in the network (A,B,C).
- Each node is a potential buyer and the value is her highest willing payment to buy the item (valuation).
- Profit of applying second price auction without promotion is 2.
- but the highest willing payment of the network is 13.

Traditional Sale Promotions

Traditional sale promotions:

- Promotions in shopping centres
- Keywords based ads via search engines such as Google
- Ads via social media such as WeChat, Facebook, Twitter

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Challenge

- The return of these promotions are unpredictable.
- The seller may LOSE from the promotions.

Tackle the Challenge

Build promotion inside the market mechanism such that

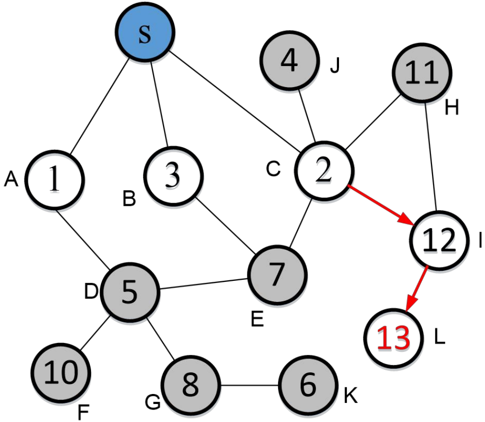
1. the promotion will never bring negative utility/revenue to the seller.
2. all buyers who are aware of the sale are incentivized to diffuse the sale information to all her neighbours.

"Diffusion Mechanism Design"

The Challenge

Why a buyer would bring more buyers to compete with her?

- Only if their efforts are rewarded!



What is Mechanism/Market Design?

- it is known as Reverse Game Theory

A Simple Mechanism Design Example

Design Goal

How can a house-seller sell her house with the "highest" profit?

A Mechanism Design Example

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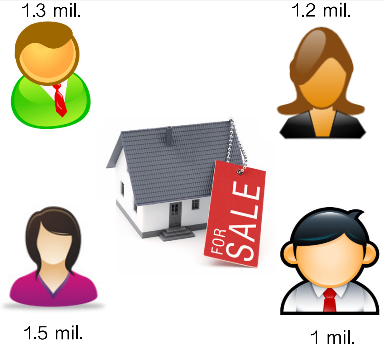

- **Challenge**: the seller doesn't know how much the buyers are willing to pay (their valuations).

A Mechanism Design Example

Design Goal

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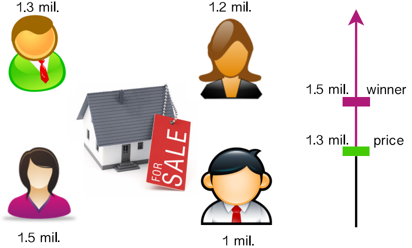

Solution: Second Price Auction (Vickrey Auction/VCG)

- Input: each buyer reports a price/bid to the seller
- Output: the seller decides
  - *allocation*: the agent with the highest price wins.
  - *payment*: the winner pays the second highest price.

A Mechanism Design Example

Design Goal

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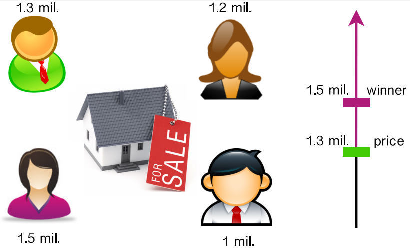

Solution: Second Price Auction (Vickrey Auction/VCG)

Properties:

- Efficient: maximising social welfare
- Truthful: buyers report their willing payments truthfully

Is this the BEST the seller can do?

Question

What can the seller do to FURTHER increase her profit?

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Question

What can the seller do to FURTHER increase her profit?

- estimate a good reserve price [Myerson 1981]
  - *requires a good estimation of buyers' valuations*
- promotions: let more people know/participate in the auction

Is this the BEST the seller can do?

Question

What can the seller do to FURTHER increase her profit?

- estimate a good reserve price [Myerson 1981]
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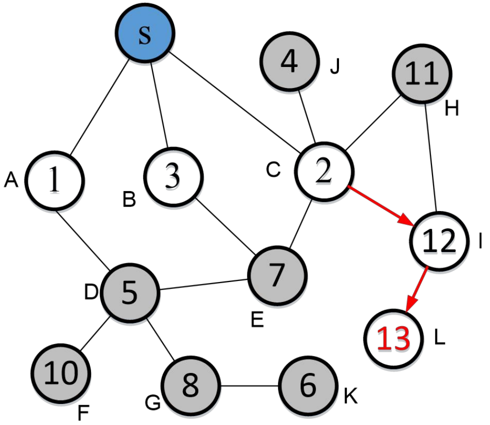
Our Solutions

Information Diffusion Mechanisms

- Dengji Zhao, Bin Li, Junping Xu, Dong Hao, Nick Jennings: *Selling Multiple Items via Social Networks*. AAMAS'18.
- Bin Li, Dong Hao, Dengji Zhao, Tao Zhou: *Mechanism Design in Social Networks*. AAAI'17.
- Bin Li, Dong Hao, Dengji Zhao, Tao Zhou: *Customer Sharing in Economic Networks with Costs*. IJCAI-ECAI'18.

Information Diffusion Paths

An information diffusion path from the seller to node L:
$s \rightarrow C \rightarrow I \rightarrow L$



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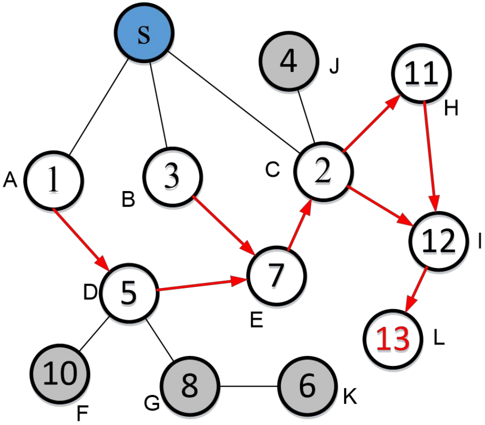
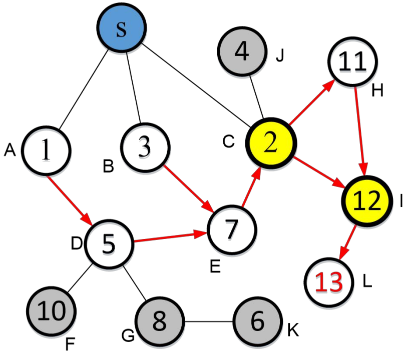


Diffusion Critical Nodes



Definition

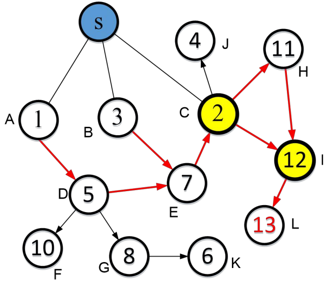
i is *j*'s diffusion critical node if all the information diffusion paths started from the seller *s* to *j* have to pass *i*.

- nodes C and I are L's only diffusion critical nodes.

The Information Diffusion Mechanism

The **payment** definition (second-price-like):

- If a buyer or one of her "*diffusion critical children*" gets the item, then the buyer pays the highest bid of the others (without the buyer's participation);
- otherwise, her payment is zero.



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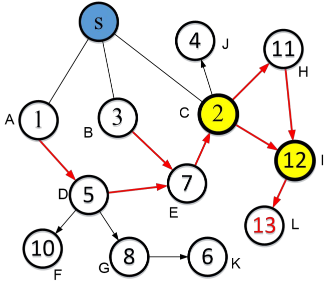


If the item is allocated to *L*, the payments of C, I and L are 10, 11, 12 respectively .

The Information Diffusion Mechanism [Li et al. AAAI'17]

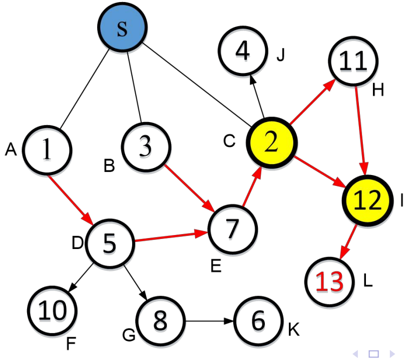
The **allocation** definition:

- Identify the node i with the highest bid and the node's diffusion critical node path $P_{c_i} = (c_i^1, c_i^2, ..., i)$.
- Give the item to the first node of P_{c_i}, the node pays to the seller and then decides to whether keep the item or pass it to the next node in P_{c_i}:
  - If the payment of the next node is greater than the bid of the current node, passes it to the next node and receives the payment from the next node; the next node makes a similar decision;
  - otherwise, keep the item.

The Information Diffusion Mechanism

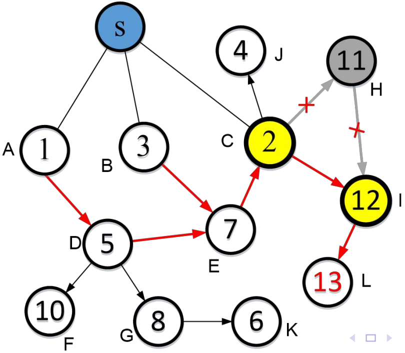
The outcome of the Information Diffusion Mechanism:

- the item is allocated to node I.
- node I pays 11 to C, C pays 10 to the seller.
- the utilities of I, C, the seller are 1, 1, 10.

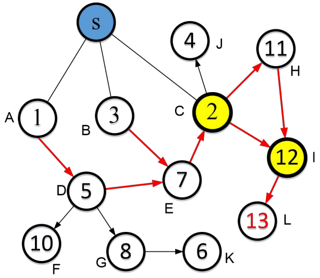


Why Buyers are Happy to Diffuse the Information?

- buyers receive the information earlier have higher priority to win the item (*C* chooses before *I* and *I* chooses before *L*).
- diffuse the information to more buyers will potentially increase their reward (if *C* does not invite *H*, her utility is 0).



Properties of the Information Diffusion Mechanism



- Truthful: report true valuation and diffuse the sale information to all her neighbours is a dominate strategy.
- Individually Rational: no buyer will receive a negative utility to join the mechanism.
- Seller's Revenue Improved: the seller's revenue is non-negative and is $\geq$ that of the VCG without diffusion.

What Next?

- Diffusion mechanisms for combinatorial exchanges
- Diffusion with costs and delays
- Network structure based revenue analysis
- Applications/implementations in the existing social networks
- Other mechanisms to further improve the revenue and/or the efficiency

Diffusion Mechanisms for Combinatorial Exchanges

Challenge

How to generalise the mechanism to combinatorial settings?

Diffusion Mechanisms for Combinatorial Exchanges

Consider the following simple setting:

- A seller sells three units of one commodity, e.g. MacBook computers.
- Each buyer has a diminishing marginal utility for consuming the goods.

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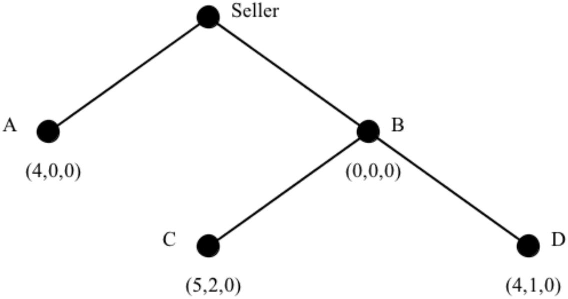
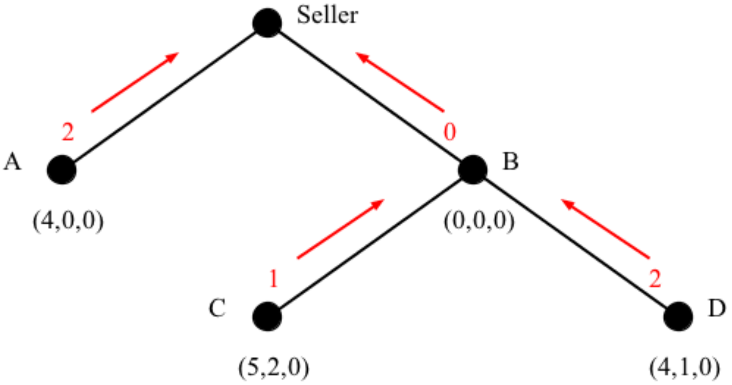


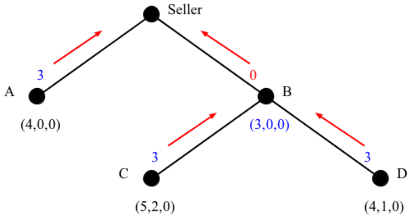
Diffusion Mechanisms for Combinatorial Exchanges

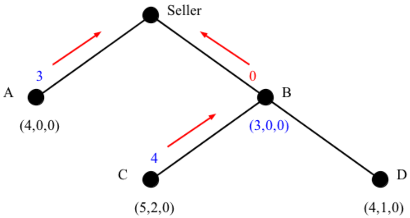
If we simply apply our information diffusion mechanism:



Diffusion Mechanisms for Combinatorial Exchanges

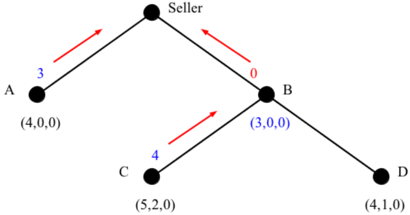
What if buyer B's valuation is $(3, 0, 0)$?

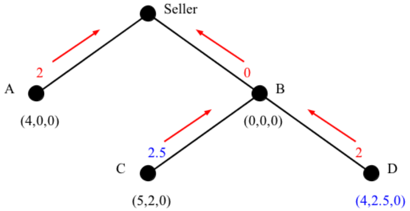




Diffusion Mechanisms for Combinatorial Exchanges

What if buyer D's valuation is $(4, 2.5, 0)$?

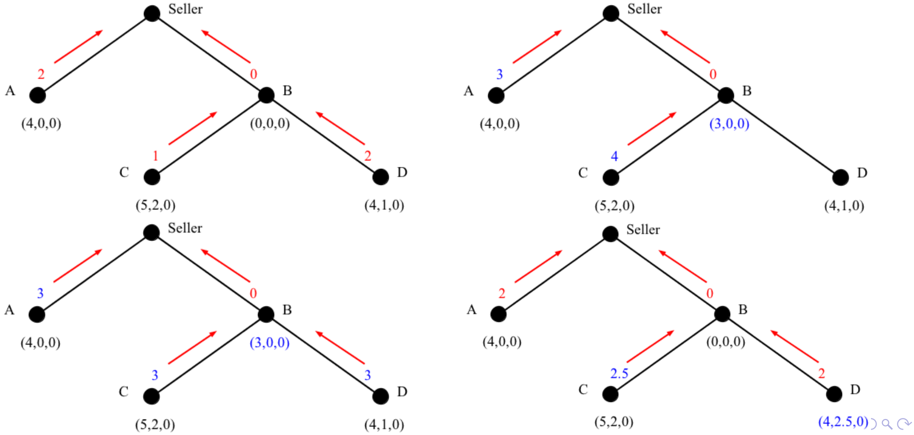




Diffusion Mechanisms for Combinatorial Exchanges

Challenge

There is a very complex **Decision Making** at each node!!!



Why is it so complex when there are multiple items?

To achieve truthfulness:

- The mechanism has to maximise each node's utility under truthful reporting/diffusing.
- Each node's payment should not depend on her valuation.

The complexity issue we had:

- A node can influence her received payments by controlling the items passed to her children.
- A node can influence the payments of her peers, without changing her own allocation and payments.
- This leads to a decision loop (very complex optimization) and may not able to maximise everyone's utility.

Reduce the Complexity

The Main Idea

A node CANNOT influence the payments she receives by controlling the items passed to her children.

Simplify the decision complexity we had:

- ~~A node can influence her received payments by controlling the items passed to her children.~~
- A node can influence the payments of her peers, without changing her own allocation and payments.
- ~~This leads to a decision loop and may not able to maximise everyone's utility.~~

Solution Example: Sells Multiple Homogeneous Items

Selling Multiple Items via Social Networks [Zhao et al. AAMAS'18]

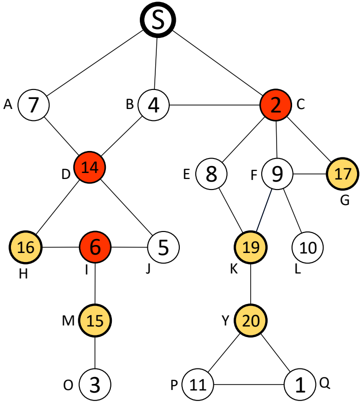
- generalised the result from [Li et al. 2017];
- agent *i*'s reward/payment doesn't depends on how many of *i*'s children received items;
- agent pays to the seller directly rather than to their parent;

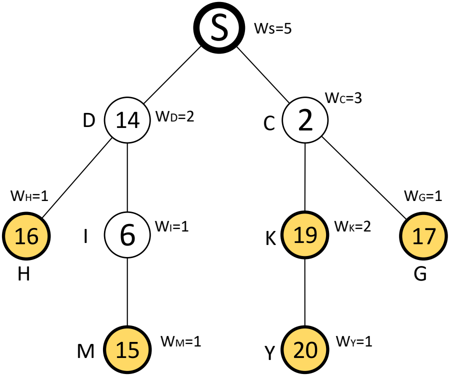
The Generalised Setting

- A seller sells $\mathcal{K} \geq 1$ homogeneous items;
- each buyer requires at most one item (single-unit demand);
- the rest is the same as [Li et al. 2017].

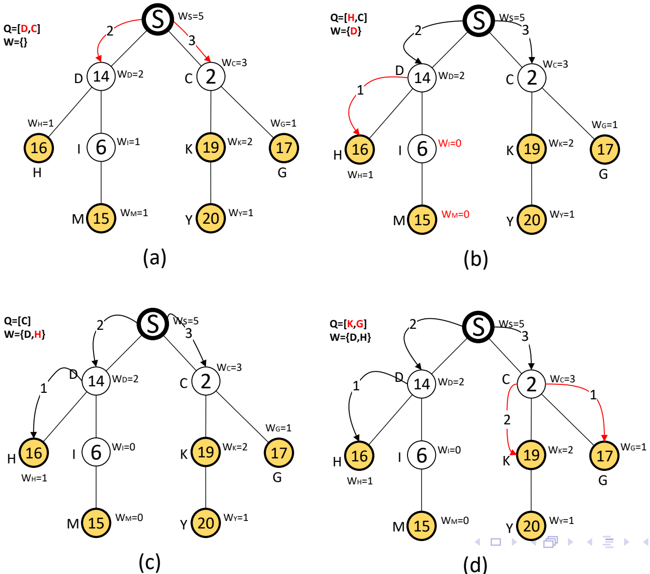
The Generalised Diffusion Mechanism

Consider $\mathcal{K} = 5$:





The Generalised Diffusion Mechanism



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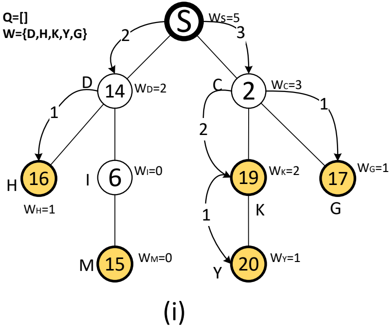
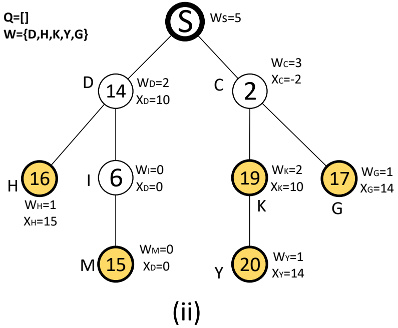




The Allocation Policy of the Generalisation

Node/buyer i receives one item if and only if

1. the top $\mathcal{K}$-highest valued children of i (and their parents, who are also i's children) do not participate
2. and i wins under the efficient allocation with their absence

given that all i's (critical) parents' allocation is determined and fixed.

The Payment Policy of the Generalisation

Node i's utility is the social welfare difference of the efficient allocation between

1. the top $\mathcal{K}$-highest valued children of i (and their parents, who are also i's children) do not participate (guarantees that i's payment does not depend on how many items i's children get)
2. and i (and all her children) does not participate

Formally, i's payment is:

$$\begin{cases} \mathcal{SW}_{-D_i} - (\mathcal{SW}_{-\mathcal{C}_i^{\mathcal{K}}} - v_i') & \text{if } i \in W, \\ \mathcal{SW}_{-D_i} - \mathcal{SW}_{-\mathcal{C}_i^{\mathcal{K}}} & \text{if } i \in \bigcup_{j \in W} \mathcal{P}_j(\theta') \setminus W, \\ 0 & \text{otherwise.} \end{cases}$$

where W is the set of nodes each of whom received one item.

Properties of the Generalisation

- Truthful: report true valuation and diffuse the sale information to all her neighbours is a dominate strategy for each node.
- Individually Rational: no node will receive a negative utility to join the mechanism.
- Seller's Revenue Improved: the seller's revenue is non-negative and is $\geq$ that of the VCG without diffusion.

Truthfulness and IR

Given i's payment:

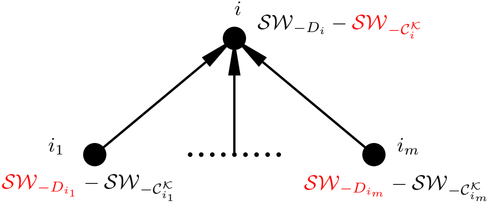
$$\begin{cases} \mathcal{SW}_{-D_i} - (\mathcal{SW}_{-\mathcal{C}_i^{\mathcal{K}}} - v_i') & \text{if } i \in W, \\ \mathcal{SW}_{-D_i} - \mathcal{SW}_{-\mathcal{C}_i^{\mathcal{K}}} & \text{if } i \in \bigcup_{j \in W} \mathcal{P}_j(\theta') \setminus W, \\ 0 & \text{otherwise.} \end{cases}$$

if i reports truthfully, i's utility is:

$$\mathcal{SW}_{-\mathcal{C}_i^{\mathcal{K}}} - \mathcal{SW}_{-D_i}$$

- $\mathcal{SW}_{-D_i}$ is the optimal social welfare without i's participation
- $\mathcal{SW}_{-\mathcal{C}_i^{\mathcal{K}}}$ is the optimal social welfare when the top $\mathcal{K}$-highest valued children of i (and their parents, who are also i's children) do not participate

Guaranteed Revenue Improvement for the Seller



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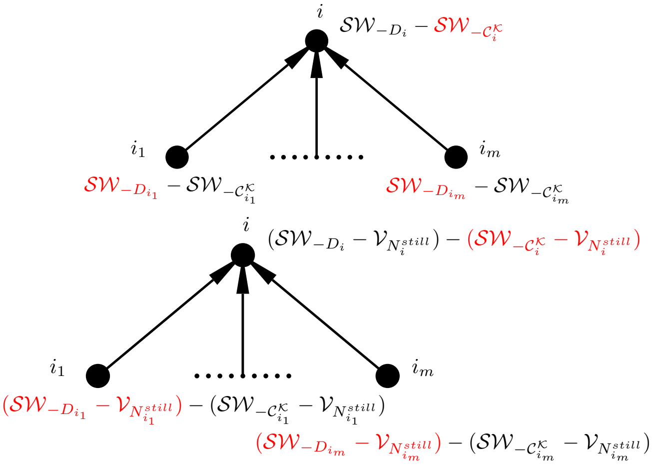

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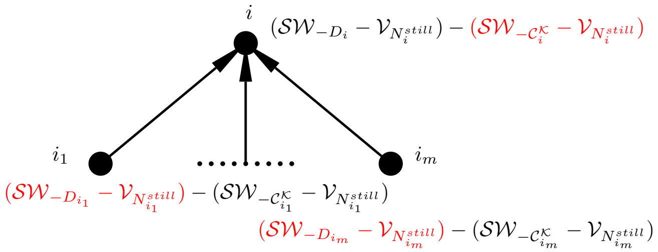
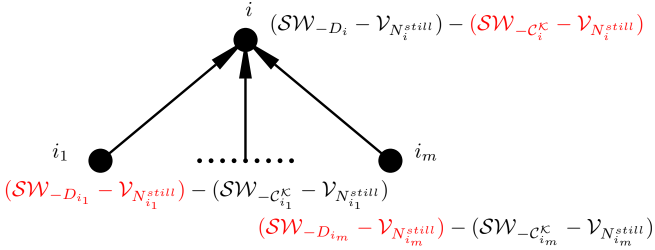


$$\mathcal{SW}_{-\mathcal{C}_i^{\mathcal{K}}} - \mathcal{V}_{N_i^{still}} \leq \sum_{i_l} (\mathcal{SW}_{-D_{i_l}} - \mathcal{V}_{N_{i_l}^{still}})$$

Guaranteed Revenue Improvement for the Seller



Theorem

The revenue of the generalised information diffusion mechanism is greater than or equal to $\mathcal{K} \times v_{\mathcal{K}+1}$, where $v_{\mathcal{K}+1}$ is the $(\mathcal{K}+1)$-th largest valuation report among r_s, assume that $|r_s| > \mathcal{K}$.

More Details

Get Confused?!

More Details

- Tutorial on 14th Morning (8:30-10:00, K11): Dengji Zhao, T26: Diffusion Mechanism Design in Social Networks.
- IJCAI, 18th 8:30-9:45: Customer Sharing in Economic Networks with Costs. [Zhao et al. IJCAI-ECAI'18]

References:

- Dengji Zhao, Bin Li, Junping Xu, Dong Hao, Nick Jennings: *Selling Multiple Items via Social Networks*. AAMAS'18.
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