Selling Multiple Items via Social Networks

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AAMAS’18
The seller (blue node) sells one item and has only three connections in the network (A,B,C).

Each node is a potential buyer and the value is her highest willing payment to buy the item (valuation).

Profit of applying second price auction without promotion is 2.

but the highest willing payment of the network is 13.
Traditional Sale Promotions

Traditional sale promotions:
- Promotions in shopping centres
- Keywords based ads via search engines such as Google
- Ads via social media such as WeChat, Facebook, Twitter
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Challenge
- The return of these promotions are unpredictable.
- The seller may LOSE from the promotions.
Tackle the Challenge

Build promotion inside the market mechanism such that

1. the promotion will never bring negative utility/revenue to the seller.

2. all buyers who are aware of the sale are incentivized to diffuse the sale information to all her neighbours.

"Diffusion Mechanism Design"
The Challenge

Why a buyer would bring more buyers to compete with her?
  - Only if their efforts are rewarded!
What is Mechanism Design

What is Mechanism/Market Design?

- it is known as Reverse Game Theory
A Simple Mechanism Design Example

Design Goal
How can a house-seller sell her house with the "highest" profit?
A Mechanism Design Example

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How can a house-seller sell her house with the "highest" profit?

**Challenge:** the seller doesn’t know how much the buyers are willing to pay (their valuations).
A Mechanism Design Example

Design Goal
How can a house-seller sell her house with the "highest" profit?

Solution: Second Price Auction (Vickrey Auction/VCG)
- **Input**: each buyer reports a price/bid to the seller
- **Output**: the seller decides
  - *allocation*: the agent with the highest price wins.
  - *payment*: the winner pays the second highest price.
A Mechanism Design Example

Design Goal
How can a house-seller sell her house with the "highest" profit?

Solution: Second Price Auction (Vickrey Auction/VCG)

Properties:
- Efficient: maximising social welfare
- Truthful: buyers report their willing payments truthfully
Is this the BEST the seller can do?

Question

What can the seller do to FURTHER increase her profit?
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- estimate a good reserve price [Myerson 1981]
  - requires a good estimation of buyers’ valuations
- promotions: let more people know/participate in the auction
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**Question**

What can the seller do to FURTHER increase her profit?

- estimate a good reserve price [Myerson 1981]
- **promotions**: let more people know/participate in the auction
Our Solutions

Information Diffusion Mechanisms

- Dengji Zhao, Bin Li, Junping Xu, Dong Hao, Nick Jennings: *Selling Multiple Items via Social Networks*. AAMAS’18.
- Bin Li, Dong Hao, Dengji Zhao, Tao Zhou: *Mechanism Design in Social Networks*. AAAI’17.
- Bin Li, Dong Hao, Dengji Zhao, Tao Zhou: *Customer Sharing in Economic Networks with Costs*. IJCAI-ECAI’18.
An information diffusion path from the seller to node L:
$s \rightarrow C \rightarrow I \rightarrow L$
An information diffusion path from the seller to node L:

\[ s \rightarrow C \rightarrow I \rightarrow L \]
**Definition**

\( i \) is \( j \)'s diffusion critical node if all the information diffusion paths started from the seller \( s \) to \( j \) have to pass \( i \).

- nodes \( C \) and \( I \) are \( L \)'s only diffusion critical nodes.
The **payment** definition (second-price-like):

- If a buyer or one of her "diffusion critical children" gets the item, then the buyer pays the **highest bid of the others** (without the buyer’s participation);
- otherwise, her payment is zero.
The information diffusion mechanism

The **payment** definition (second-price-like):

- If a buyer or one of her "**diffusion critical children**" gets the item, then the buyer pays the highest bid of the others (without the buyer’s participation);
- otherwise, her payment is zero.

If the item is allocated to $L$, the payments of $C$, $I$ and $L$ are **10, 11, 12** respectively.
The allocation definition:

- Identify the node $i$ with the highest bid and the node's diffusion critical node path $P_{c_i} = (c_i^1, c_i^2, \ldots, i)$.
- Give the item to the first node of $P_{c_i}$, the node pays to the seller and then decides to whether keep the item or pass it to the next node in $P_{c_i}$:
  - If the payment of the next node is greater than the bid of the current node, passes it to the next node and receives the payment from the next node; the next node makes a similar decision;
  - otherwise, keep the item.
The outcome of the Information Diffusion Mechanism:

- the item is allocated to node I.
- node I pays 11 to C, C pays 10 to the seller.
- the utilities of I, C, the seller are 1, 1, 10.
Why Buyers are Happy to Diffuse the Information?

- buyers receive the information earlier have higher priority to win the item (C chooses before I and I chooses before L).
- diffuse the information to more buyers will potentially increase their reward (if C does not invite H, her utility is 0).
Properties of the Information Diffusion Mechanism

- **Truthful**: report true valuation and diffuse the sale information to all her neighbours is a dominate strategy.
- **Individually Rational**: no buyer will receive a negative utility to join the mechanism.
- **Seller’s Revenue Improved**: the seller’s revenue is non-negative and is $\geq$ that of the VCG without diffusion.
What Next?

- Diffusion mechanisms for combinatorial exchanges
- Diffusion with costs and delays
- Network structure based revenue analysis
- Applications/implementations in the existing social networks
- Other mechanisms to further improve the revenue and/or the efficiency
Challenge

How to generalise the mechanism to combinatorial settings?
Consider the following simple setting:

- A seller sells three units of one commodity, e.g. MacBook computers.
- Each buyer has a diminishing marginal utility for consuming the goods.
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Diffusion Mechanisms for Combinatorial Exchanges

If we simply apply our information diffusion mechanism:

![Diagram showing diffusion mechanisms](image)
What if buyer B’s valuation is \((3, 0, 0)\)?
What if buyer D’s valuation is \((4, 2.5, 0)\)?
Challenge

There is a very complex **Decision Making** at each node!!!
Why is it so complex when there are multiple items?

To achieve truthfulness:
- The mechanism has to **maximise each node’s utility** under truthful reporting/diffusing.
- Each node’s **payment** should **not depend on her valuation**.

The complexity issue we had:
- A node can **influence her received payments** by controlling the items passed to her children.
- A node can **influence the payments of her peers**, without changing her own allocation and payments.
- **This leads to a decision loop (very complex optimization)** and may not able to maximise everyone’s utility.
The Main Idea

A node CANNOT influence the payments she receives by controlling the items passed to her children.

Simplify the decision complexity we had:

- A node can influence her received payments by controlling the items passed to her children.
- A node can influence the payments of her peers, without changing her own allocation and payments.
- This leads to a decision loop and may not able to maximise everyone’s utility.
Solution Example: Sells Multiple Homogeneous Items

*Selling Multiple Items via Social Networks* [Zhao et al. AAMAS’18]

- generalised the result from [Li et al. 2017];
- agent $i$’s reward/payment doesn’t depend on how many of $i$’s children received items;
- agent pays to the seller directly rather than to their parent;
The Generalised Setting

- A seller sells $\mathcal{K} \geq 1$ homogeneous items;
- each buyer requires at most one item (single-unit demand);
- the rest is the same as [Li et al. 2017].
The Generalised Diffusion Mechanism

Consider $K = 5$:
The Generalised Diffusion Mechanism

(a) $Q=[D,C]$, $W=\emptyset$

(b) $Q=[H,C]$, $W=\{D\}$

(c) $Q=[C]$, $W=\{D,H\}$

(d) $Q=[K,G]$, $W=\{D,H\}$
The Generalised Diffusion Mechanism

\[ Q = \{ \} \]
\[ W = \{ D, H, K, Y, G \} \]

(i) \[
\begin{align*}
S & \quad \text{Ws} = 5 \\
D & \quad \text{W} = 2 \\
C & \quad \text{Wc} = 3 \\
H & \quad \text{Wh} = 1 \\
I & \quad \text{W} = 0 \\
M & \quad \text{Wm} = 0 \\
K & \quad \text{Wk} = 2 \\
G & \quad \text{Wg} = 1 \\
16 & \\
6 & \\
15 & \\
20 & \\
\end{align*}
\]

(ii) \[
\begin{align*}
S & \quad \text{Ws} = 5 \\
D & \quad \text{W} = 2 \quad \text{Xd} = 10 \\
C & \quad \text{Wc} = 3 \quad \text{Xc} = -2 \\
H & \quad \text{Wh} = 1 \quad \text{Xh} = 15 \\
I & \quad \text{W} = 0 \quad \text{Xd} = 0 \\
M & \quad \text{Wm} = 0 \quad \text{Xd} = 0 \\
K & \quad \text{Wk} = 2 \quad \text{Xk} = 10 \\
G & \quad \text{Wg} = 1 \quad \text{Xg} = 14 \\
16 & \\
6 & \\
15 & \\
20 & \\
\end{align*}
\]
Node/buyer $i$ receives one item if and only if

1. the top $\mathcal{K}$-highest valued children of $i$ (and their parents, who are also $i$’s children) do not participate

2. and $i$ wins under the efficient allocation with their absence given that all $i$’s (critical) parents’ allocation is determined and fixed.
The Payment Policy of the Generalisation

Node i’s utility is the social welfare difference of the efficient allocation between
1. the top $\kappa$-highest valued children of i (and their parents, who are also i’s children) do not participate (guarantees that i’s payment does not depend on how many items i’s children get)
2. and i (and all her children) does not participate

Formally, i’s payment is:

$$\begin{align*}
&\begin{cases}
SW_{-D_i} - (SW_{-C_i^\kappa} - v'_i) & \text{if } i \in W, \\
SW_{-D_i} - SW_{-C_i^\kappa} & \text{if } i \in \bigcup_{j \in W} P_j(\theta') \setminus W, \\
0 & \text{otherwise.}
\end{cases}
\end{align*}$$

where $W$ is the set of nodes each of whom received one item.
Properties of the Generalisation

- **Truthful**: report true valuation and **diffuse the sale information to all her neighbours** is a dominate strategy for each node.
- **Individually Rational**: no node will receive a negative utility to join the mechanism.
- ** Seller’s Revenue Improved**: the seller’s revenue is non-negative and is $\geq$ that of the VCG without diffusion.
Truthfulness and IR

Given $i$’s payment:

$$
\begin{cases}
    SW_{-D_i} - (SW_{-c_i^K} - v_i') & \text{if } i \in W, \\
    SW_{-D_i} - SW_{-c_i^K} & \text{if } i \in \bigcup_{j \in W} \mathcal{P}_j(\theta') \setminus W, \\
    0 & \text{otherwise}.
\end{cases}
$$

if $i$ reports truthfully, $i$’s utility is:

$$SW_{-c_i^K} - SW_{-D_i}$$

- $SW_{-D_i}$ is the optimal social welfare without $i$’s participation
- $SW_{-c_i^K}$ is the optimal social welfare when the top $K$-highest valued children of $i$ (and their parents, who are also $i$’s children) do not participate
Guaranteed Revenue Improvement for the Seller

Abstract

We consider dual-role exchange markets, where traders can offer to both buy and sell the same commodity in the exchange but, if they transact, they can only be either a buyer or all else, which is determined by the market mechanism. To design desirable mechanisms for such exchanges, we show that existing solutions may not be incentive compatible, and importantly, cause the market maker to suffer a significant deficit. Hence, to combat this problem, following McAfee’s trade reduction approach, we propose a new trade reduction mechanism, called balanced trade reduction, that is incentive compatible and also provides flexible trade-offs between efficiency and deficit.
Guaranteed Revenue Improvement for the Seller

\[ SW_{D_i} - SW_{c_i^k} \]

\[ (SW_{D_{i_1}} - V_{N_{i_1}^{still}}) - (SW_{c_{i_1}^k} - V_{N_{i_1}^{still}}) \]

\[ (SW_{D_{i_m}} - V_{N_{i_m}^{still}}) - (SW_{c_{i_m}^k} - V_{N_{i_m}^{still}}) \]
Guaranteed Revenue Improvement for the Seller

Abstract

We consider dual-role exchange markets, where traders can offer to both buy and sell the same commodity in the exchange but, if they transact, they can only be either a buyer or a seller, which is determined by the market mechanism. To design desirable mechanisms for such exchanges, we show that existing solutions may not be incentive compatible, and importantly, cause the market maker to suffer a significant deficit. Hence, to combat this problem, following McAfee’s trade reduction approach, we propose a new trade reduction mechanism, called balanced trade reduction, that is incentive compatible and also provides flexible trade-offs between efficiency and deficit.
Guaranteed Revenue Improvement for the Seller

Balanced Trade Reduction for Dual-Role Exchange Markets

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Abstract

We consider dual-role exchange markets, where traders can offer to both buy and sell the same commodity in the exchange but, if they transact, they can only be either a buyer or as seller, which is determined by the market mechanism. To design desirable mechanisms for such exchanges, we show that existing solutions may not be incentive compatible, and importantly, cause the market maker to suffer a significant deficit. Hence, to combat this problem, following McAfee’s trade reduction approach, we propose a new trade reduction mechanism, called balanced trade reduction, that is incentive compatible and also provides flexible trade-offs between efficiency and deficit.

Theorem

The revenue of the generalised information diffusion mechanism is greater than or equal to $K \times v_{K+1}$, where $v_{K+1}$ is the $(K + 1)$-th largest valuation report among $r_s$, assume that $|r_s| > K$.
Get Confused?!
More Details

- **Tutorial on 14th Morning (8:30-10:00, K11):** Dengji Zhao, T26: Diffusion Mechanism Design in Social Networks.

- **IJCAI, 18th 8:30-9:45:** Customer Sharing in Economic Networks with Costs. [Zhao et al. IJCAI-ECAI’18]

**References:**

- Dengji Zhao, Bin Li, Junping Xu, Dong Hao, Nick Jennings: *Selling Multiple Items via Social Networks*. AAMAS’18.

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