Incentives for Early Arrival in Cooperative Games

(AAMAS24 Best Paper)

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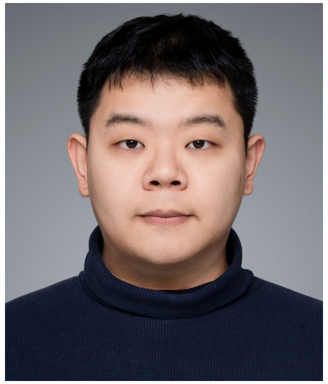
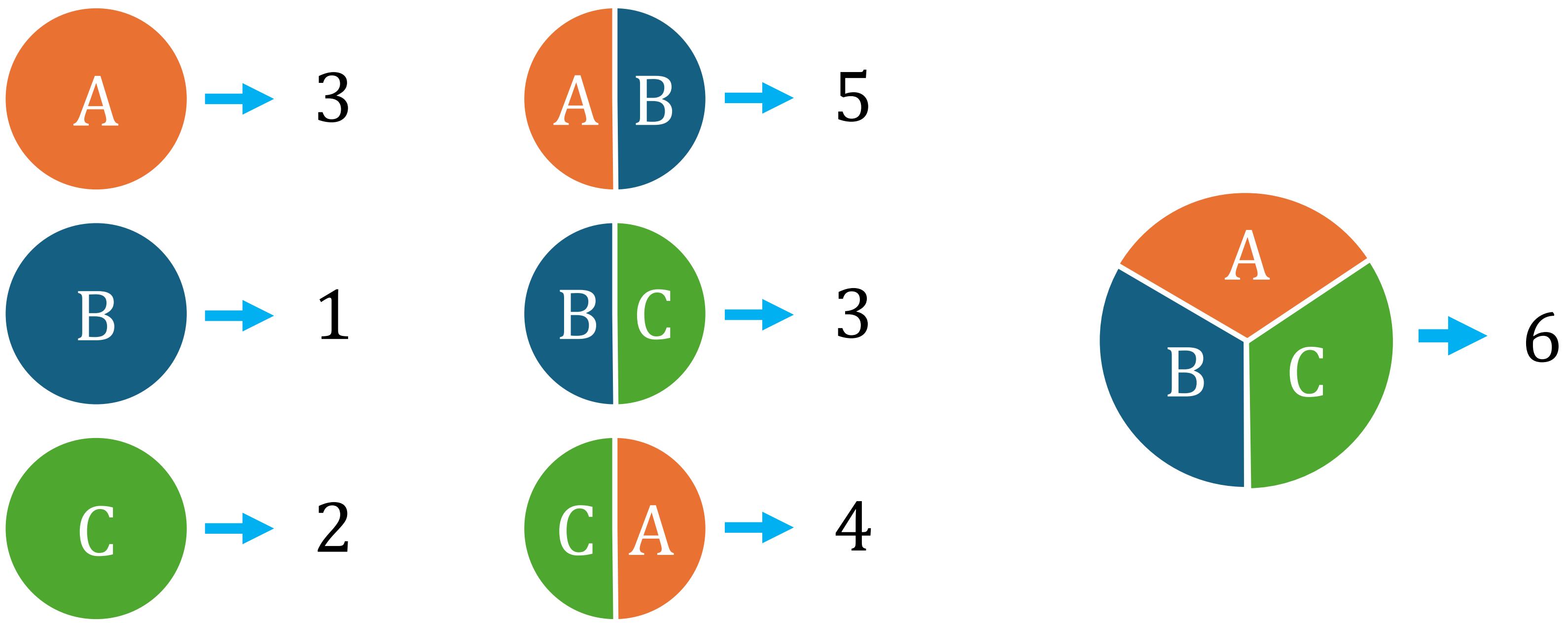






Cooperative Game

- Players cooperate to create different values



- Determine how to share the value:

  Shapley value, core, ...

Cooperative Game and Shapley Value

- Players: $N = \{A, B, C\}$
- Valuation Function: $v: 2^N \to \mathbf{R}$
- Marginal Contribution: $\mathrm{MC}(i, v, S) = v(S) - v(S \setminus i), \forall i \in S$

- Shapley Value:

$$\mathrm{SV}_i(v) \coloneqq \frac{1}{|N|!} \sum_{S \subseteq N \setminus \{i\}} |S|!\,(|N| - |S| - 1)\mathrm{MC}(i, v, S \cup \{i\})$$

The averaged marginal contribution on all possible joining orders.

Shapley Value

- SV: Averaged MC on all orders
- E.g.

Coalition	A	B	C	AB	AC	BC	ABC
Value	3	1	2	5	4	3	6

C	A	B
2	2	2

B	C	A
1	2	3

A	B	C
3	2	1

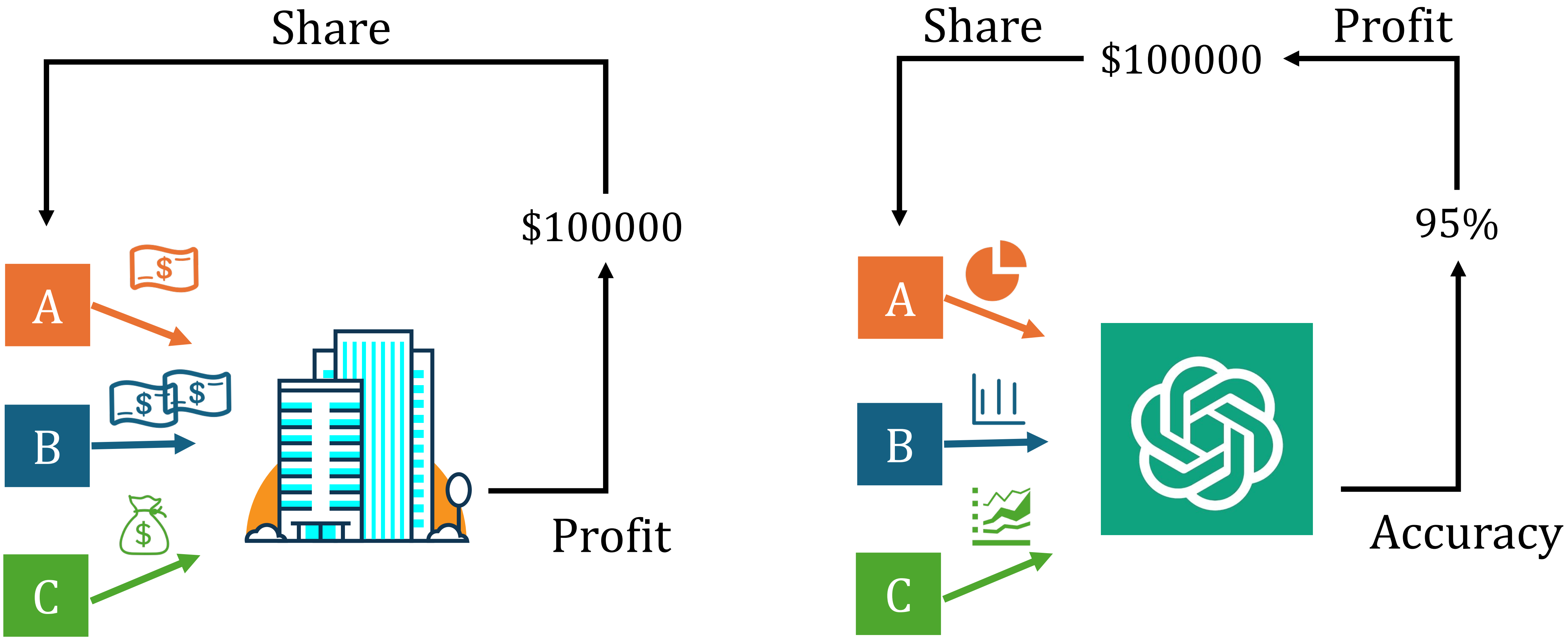
A	C	B
3	1	2

C	B	A
2	1	3

B	A	C
1	4	1

$$SV_A = \frac{2+3+3+3+3+4}{6} = 3$$

Venture Capital / Data Acquisition: Join Order Matters



Online Cooperative Games

Joining Order	A	B	C	Unknown
Value after joining	3	5	6	…
Marginal Contribution (MC)	**3**	**2**	**1**	…
The Value Share				**?**

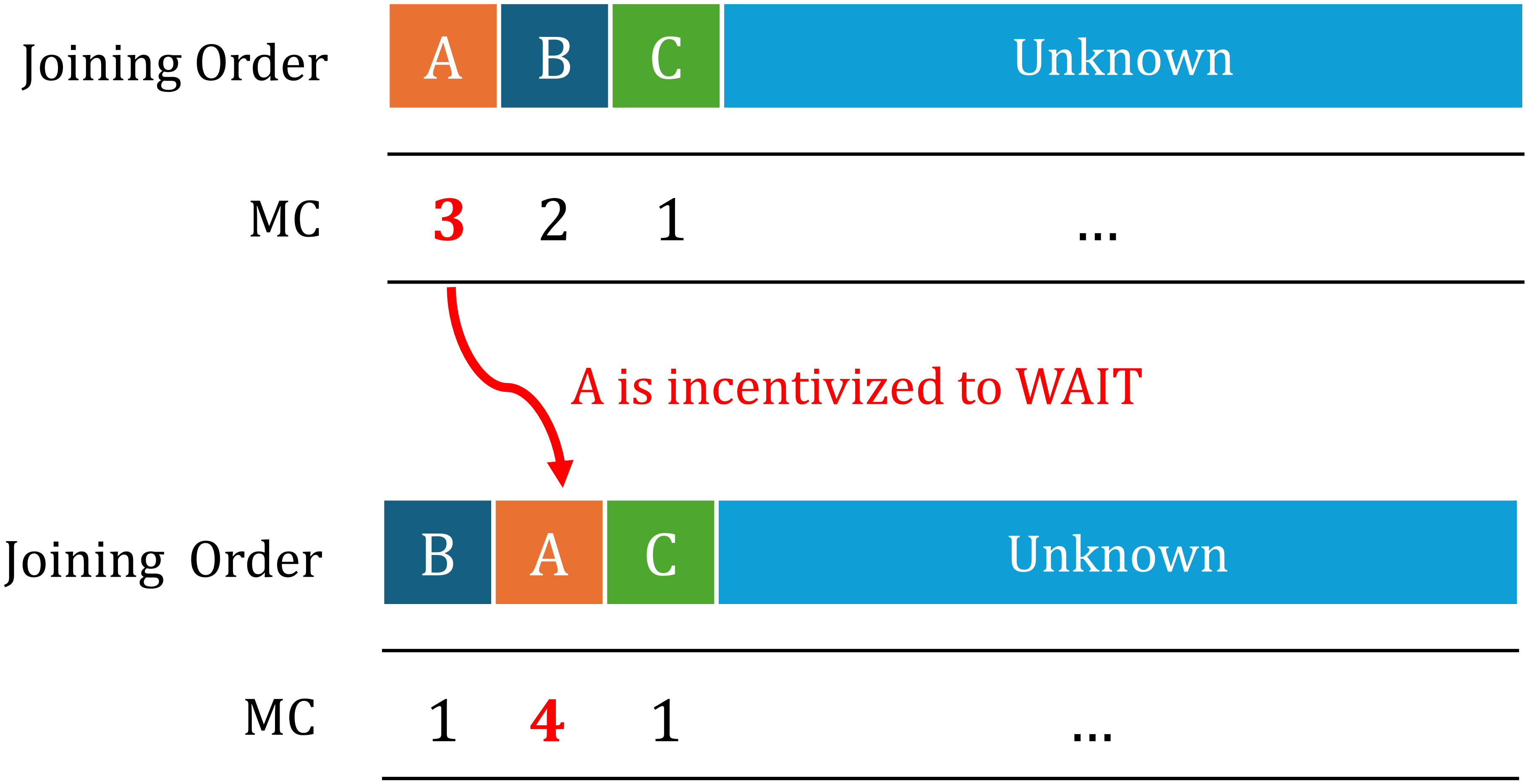
Online Cooperative Game Model

- Players: $N = \{A, B, C\}$
- Valuation Function: $v: 2^N \to \mathbf{R}$
- Joining Order: $\pi \in \Pi(N)$ (a permutation of players)
- Marginal Contribution: $\mathrm{MC}(i, v, S) = v(S) - v(S \setminus i), \forall i \in S$

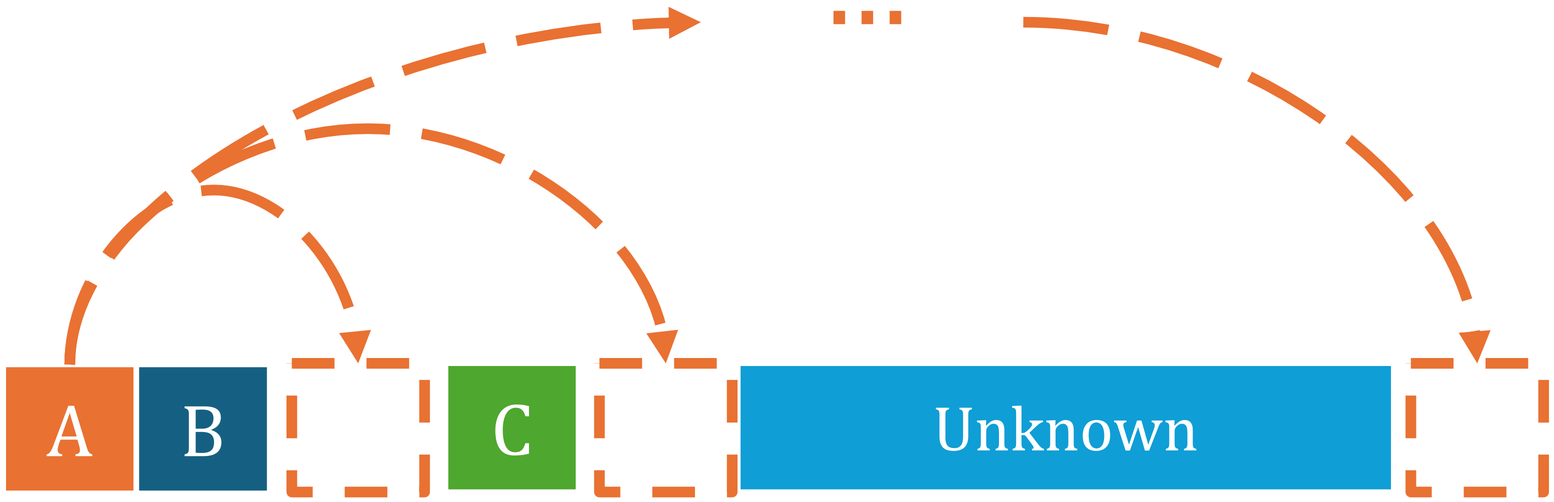
- Shapley Value:

$$\mathrm{SV}_i(v) := \frac{1}{|N|!} \sum_{S \subseteq N \setminus \{i\}} |S|!\,(|N| - |S| - 1)\mathrm{MC}(i, v, S \cup \{i\})$$

How to share the value: *Marginal Contribution*

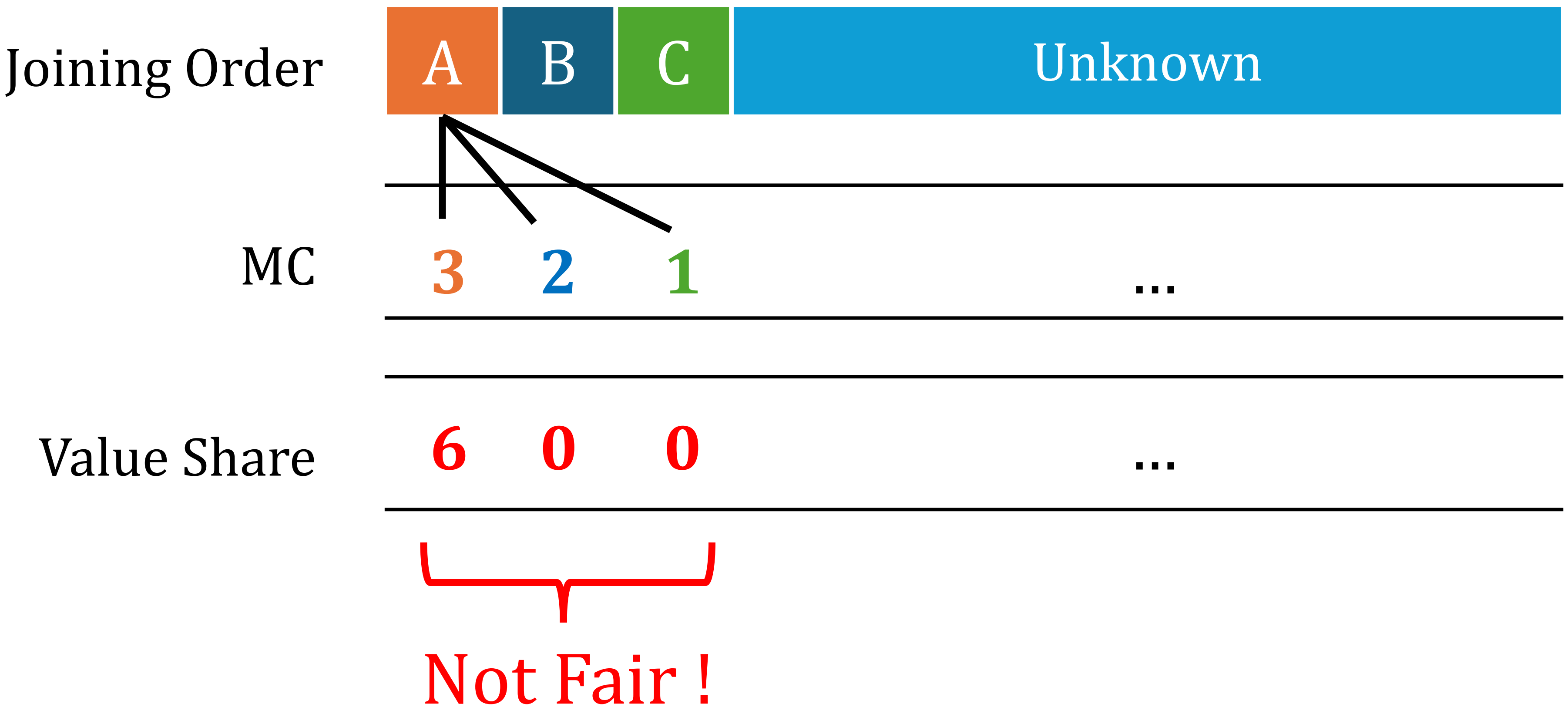


Incentivizing for Early Arrival (I4EA)

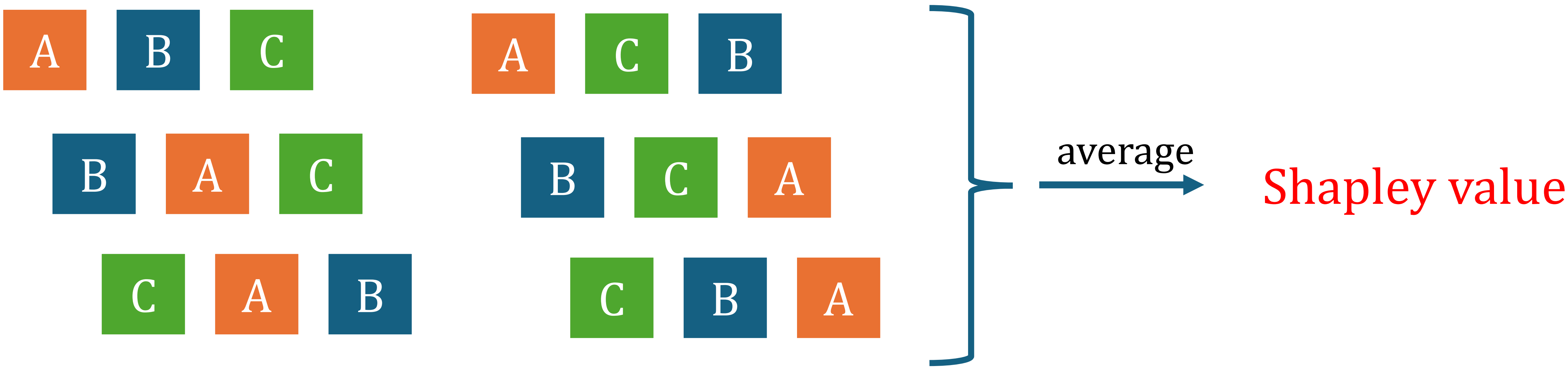


I4EA: When the order of others are fixed, the players are *incentivized to join as soon as possible*.

One Solution: *The First Gets All*

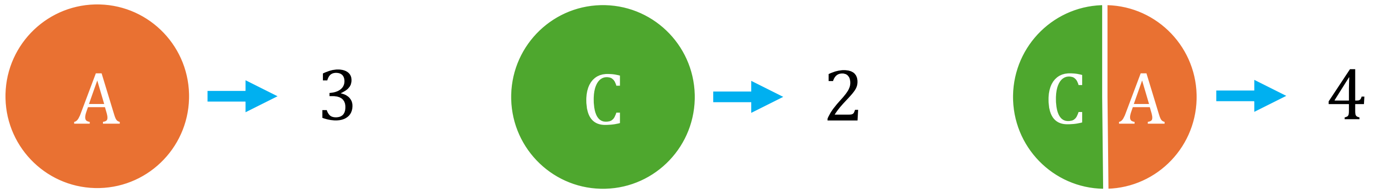


One Fairness: Shapley Fair (SF)



SF: The expected share to a player equals her *Shapley value.*

Shapley value?



Order: A | Unknown

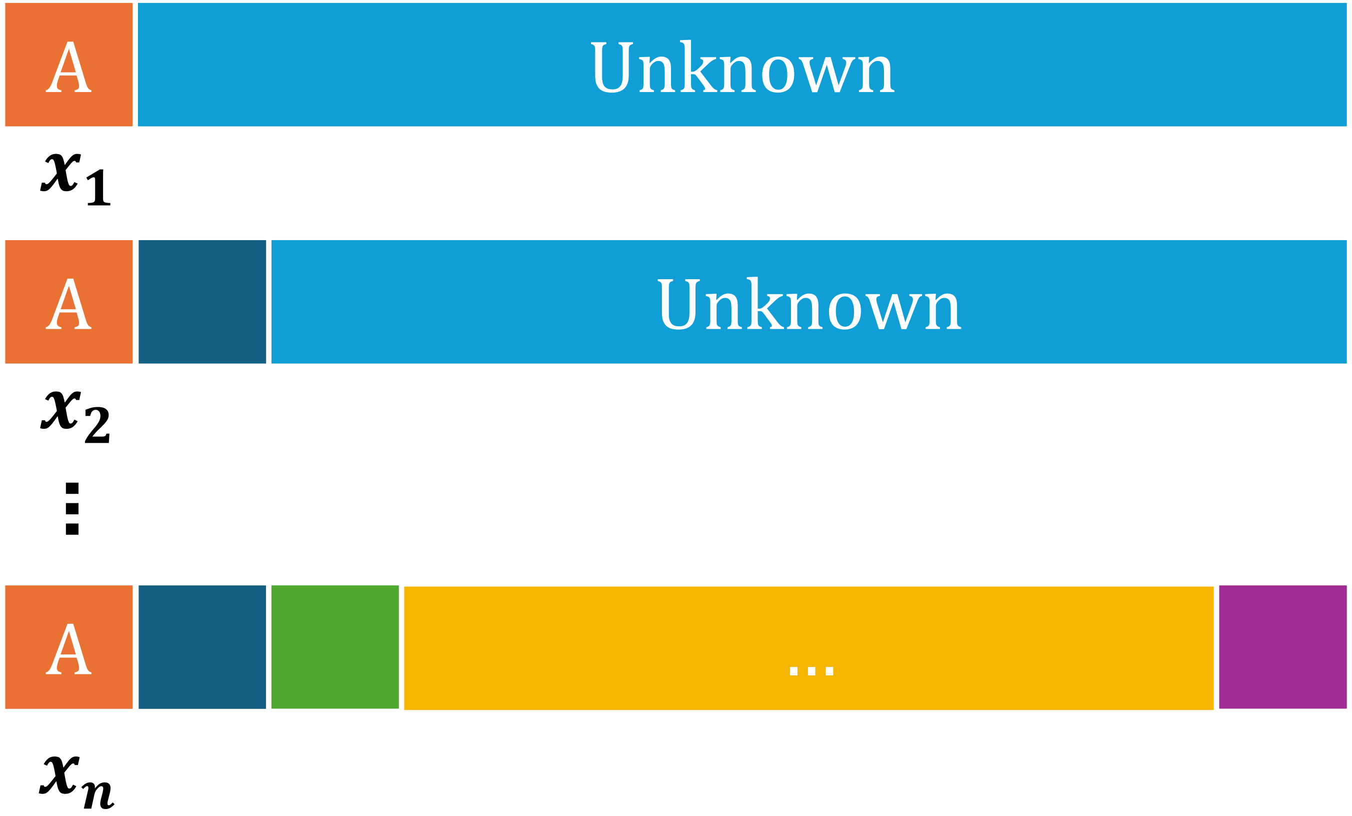
Shapley value: **3** ...

Order: A | C | Unknown

Shapley value: **2.5** **1.5** ...

A's share Decreases!

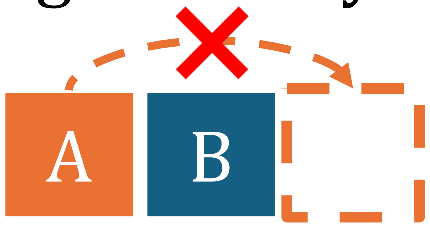
Online Individual Rational (OIR)



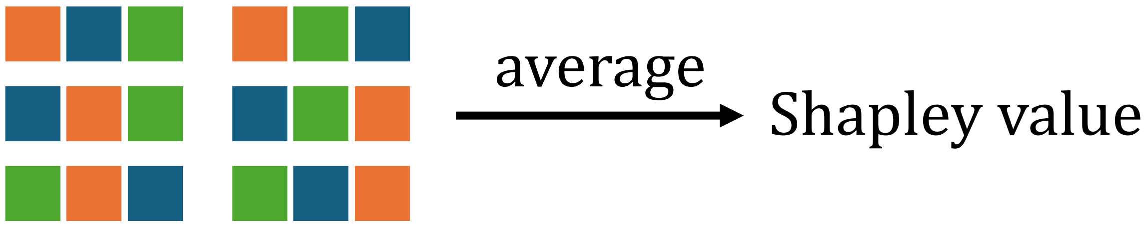
OIR: $x_n \geq \cdots \geq x_2 \geq x_1 \geq 0$ *(non-decreasing, non-negative)*

All the Desire Properties

- Incentivizing For Early Arrival (I4EA)

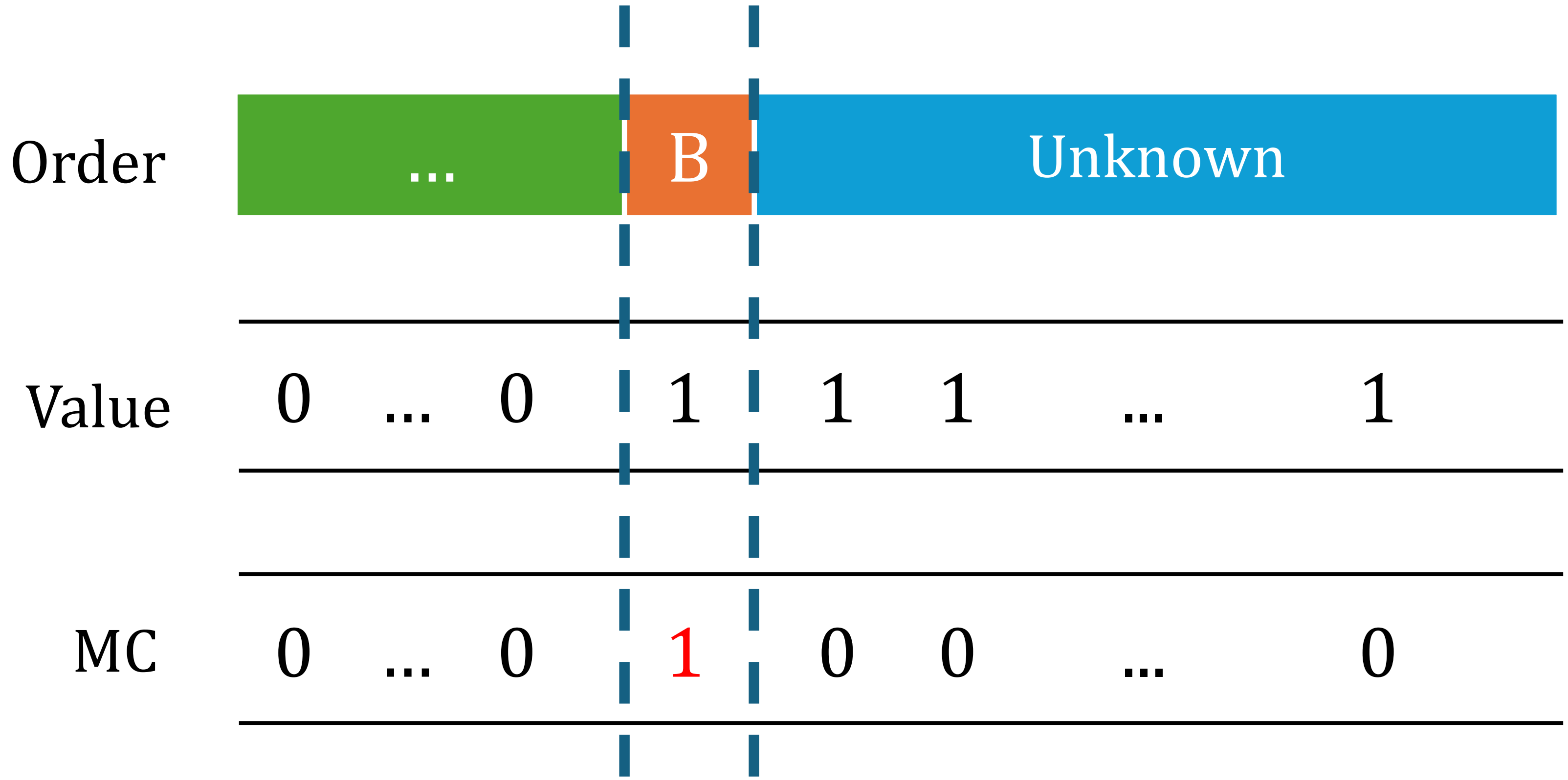


- Shapley-fair (SF)



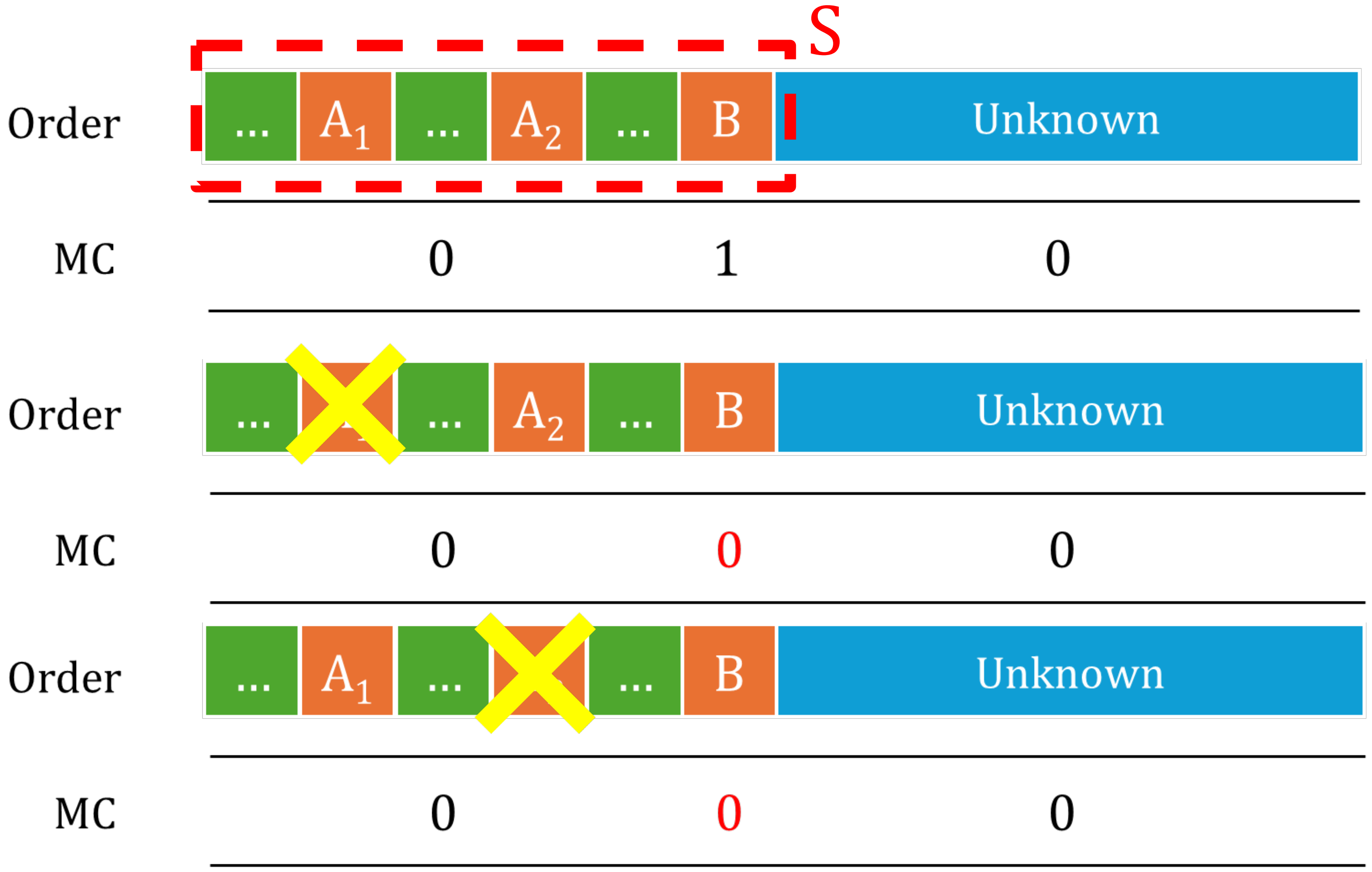
- Online Individual Rational (OIR)

Start with 0-1 Valued Monotone Games: $A_1 \wedge A_2 \wedge B$



Marginal Player: B is the only player who creates a MC of 1.

0-1 Valued Monotone Games: $A_1 \wedge A_2 \wedge B$

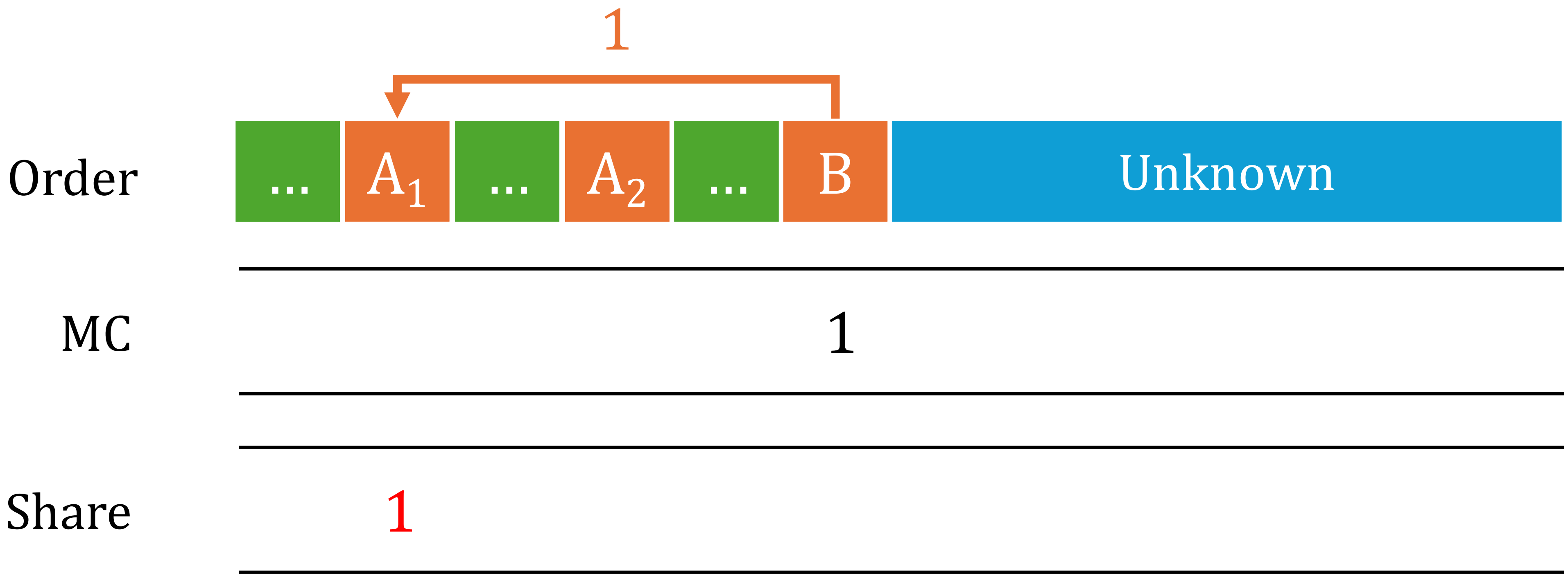


Critical Players: $\{i \mid v(S \setminus \{i\}) = 0\}$

Reward First Critical Player (RFC)

Definition: RFC

Give the MC of the marginal player to the first critical player in S.



Properties Overview

I4EA + SF + OIR → (satisfiable) → RFC is one of the solutions

given a game → (satisfiable?) → we know

Properties of RFC

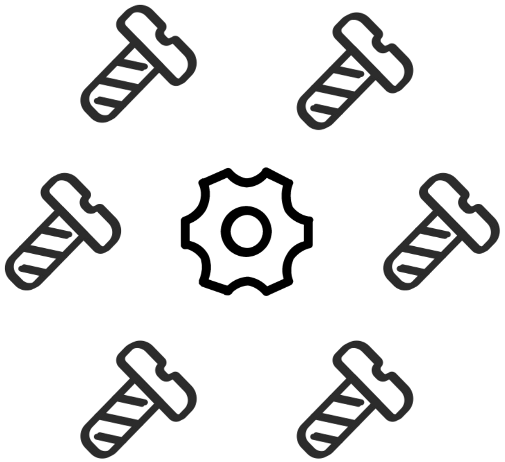
Theorem

RFC is SF and OIR on every 0-1 valued monotone game.

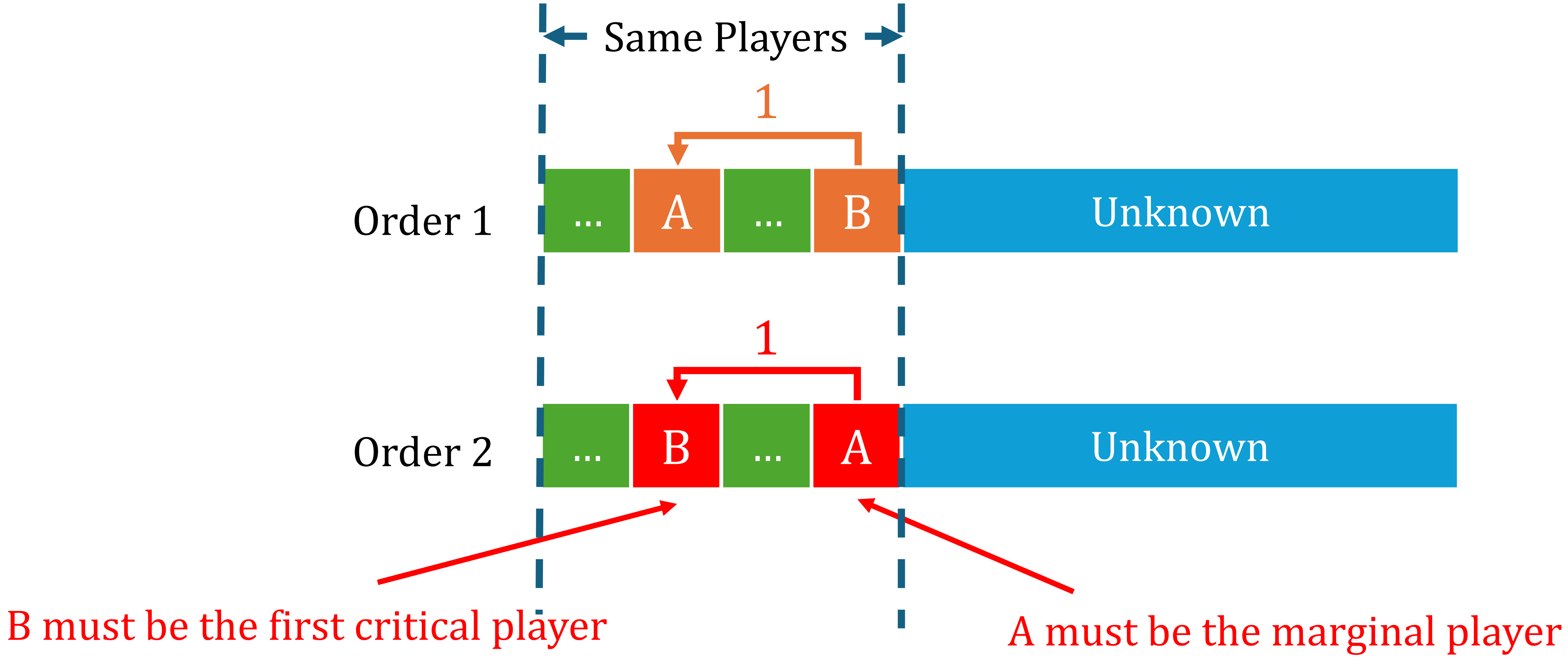
RFC is not I4EA only on games satisfying: $\exists i, v(i) = 0$ and $\exists S \subseteq N, S^* = \{i\}$. Here $S^* := \{i \in S \mid v(S) = 1,\ v(S \setminus \{i\}) = 0\}$.

- SF ✓
- OIR ✓ (obviously)
- I4EA is not satisfied when someone can delay to be the only critical player.

  e.g. $(A \vee B) \wedge C$, $\big((A \wedge B) \vee (E \wedge D)\big) \wedge C$, ...



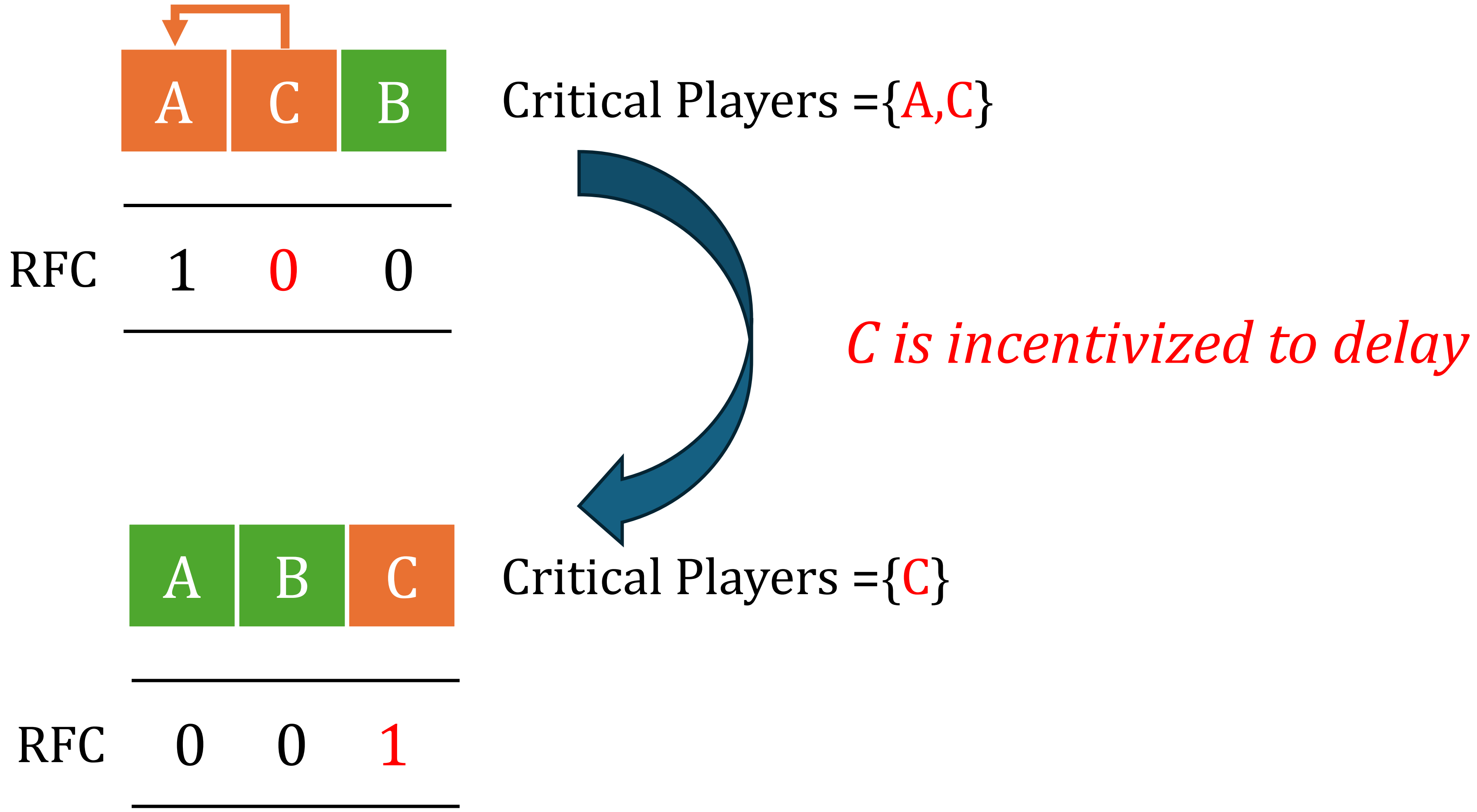
Proof of Shapley-Fair (sketch)



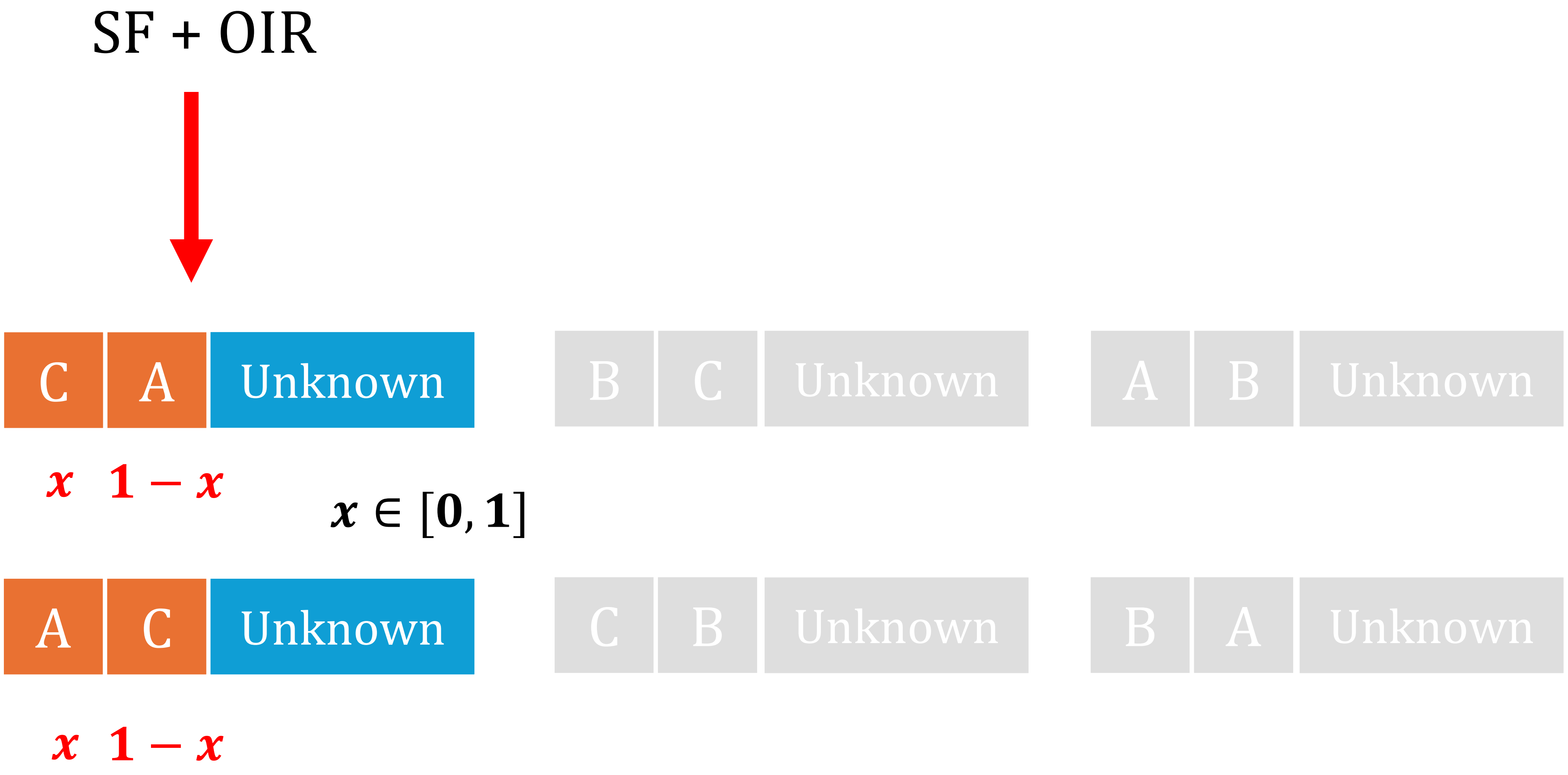
RFC on 3-player 0-1 valued monotone games

v	**Value Receiver**	**I4EA**
A	to A	Yes
$A \vee \mathrm{B}$	1^{st} of $\{A, B\}$	Yes
$A \vee B \vee C$	1^{st}	Yes
$A \wedge B$	1^{st} of $\{A, B\}$	Yes
$A \wedge B \wedge C$	1^{st}	Yes
$(\mathrm{A} \wedge B) \vee C$	C is 1^{st} or 2^{nd} $\rightarrow$ C Otherwise $\rightarrow$ 1^{st} of {A,B}	Yes
$(A \vee B) \wedge C$	**C is 1^{st} or 3^{rd} $\rightarrow$ C** **Otherwise $\rightarrow$ 1st of {A,B}**	**No**
$(A \wedge B) \vee (A \wedge C) \vee (\mathrm{B} \wedge C)$	1^{st}	Yes

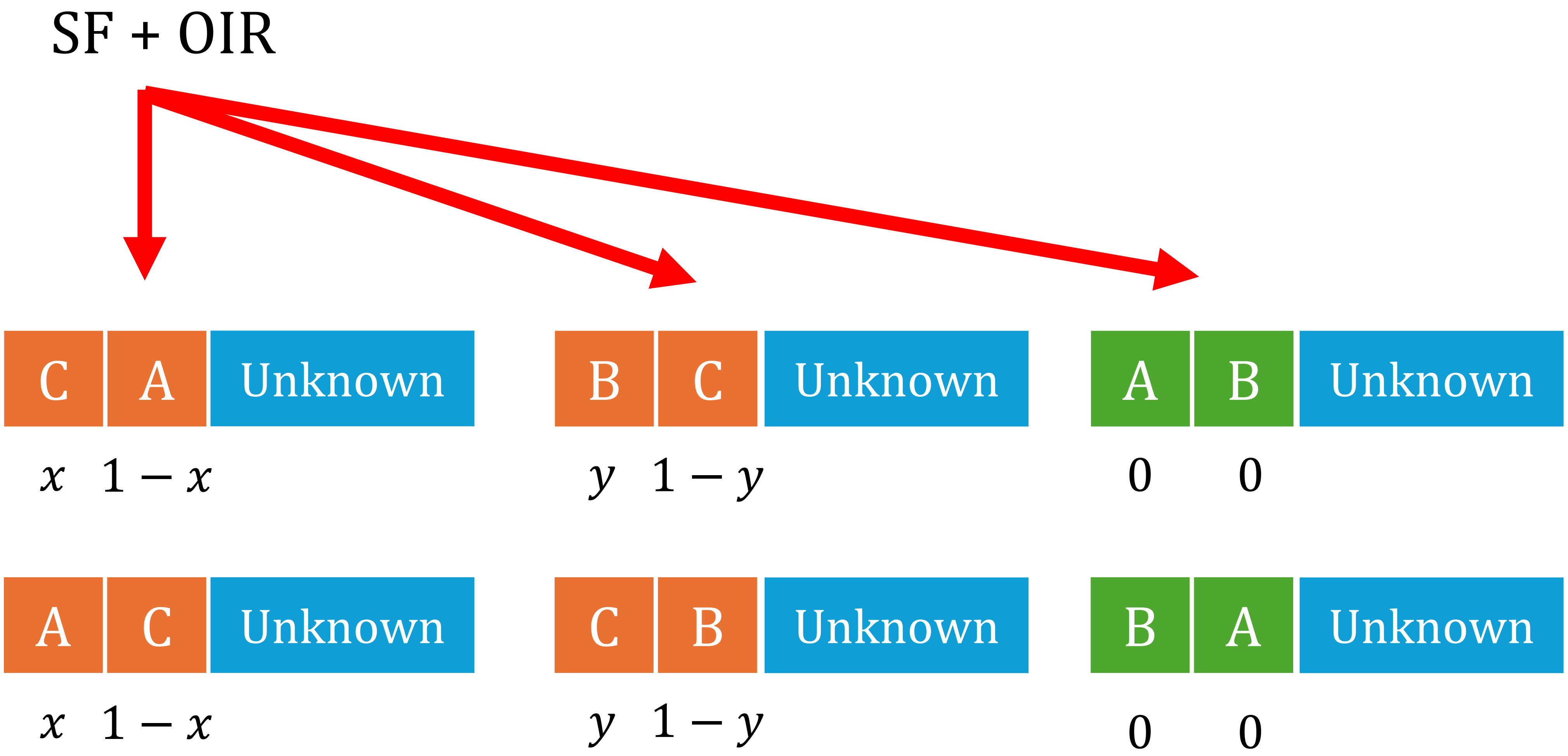
RFC is not I4EA on $(A \vee B) \wedge C$



$(A \lor B) \land C$ is unsolvable

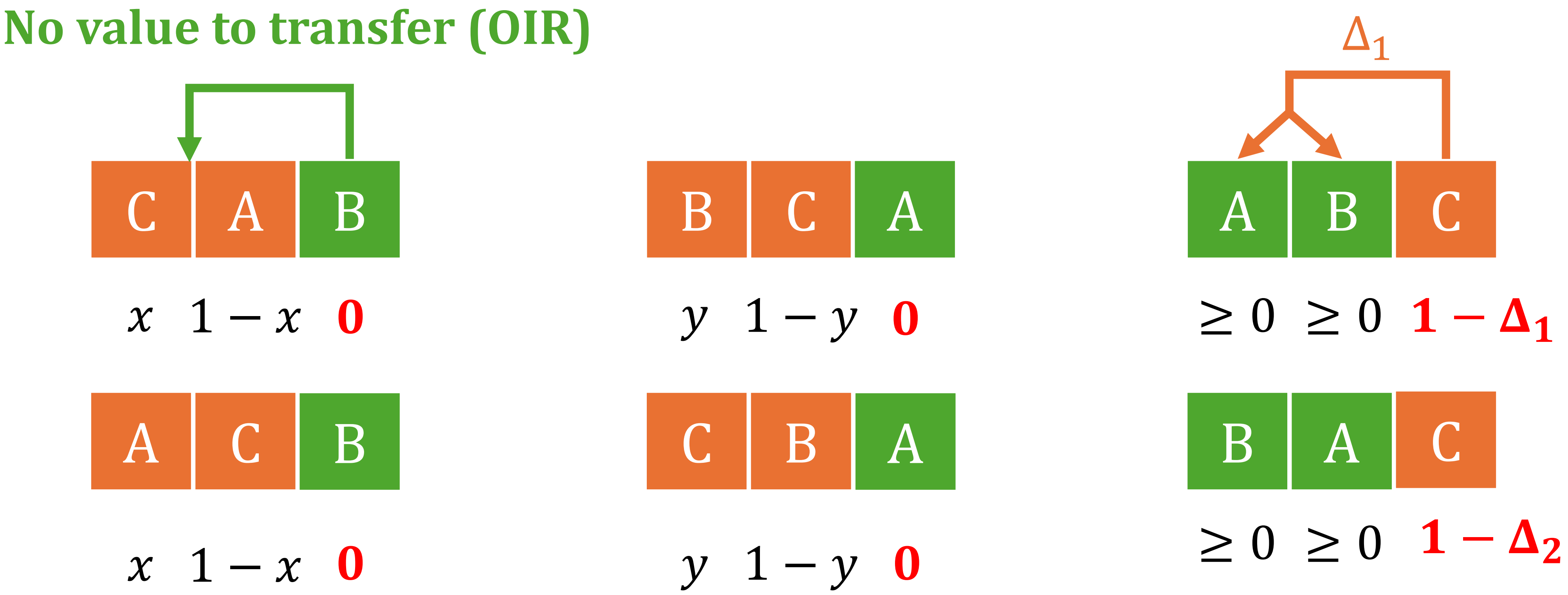


$(A \vee B) \wedge C$ is unsolvable



$(A \vee B) \wedge C$ is unsolvable

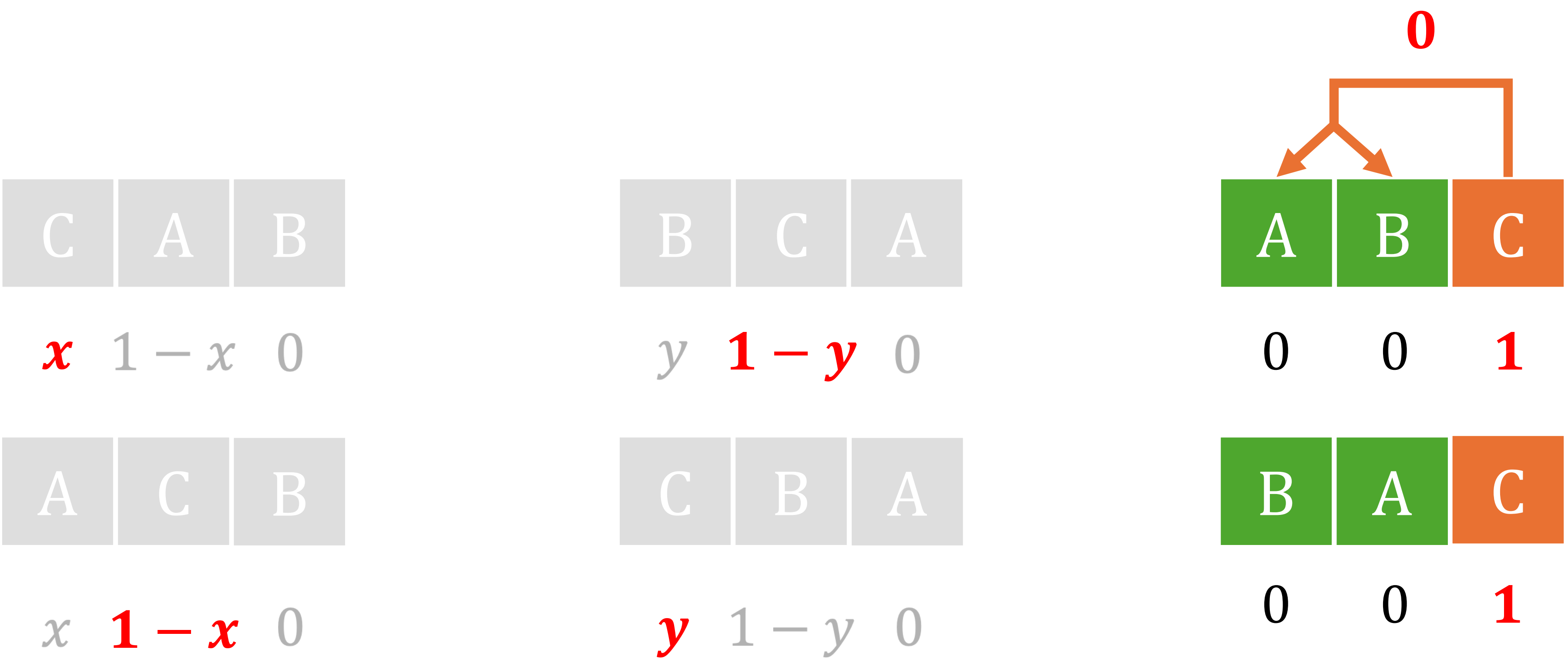
The 3rd player joins…



$(A \lor B) \land C$ is unsolvable

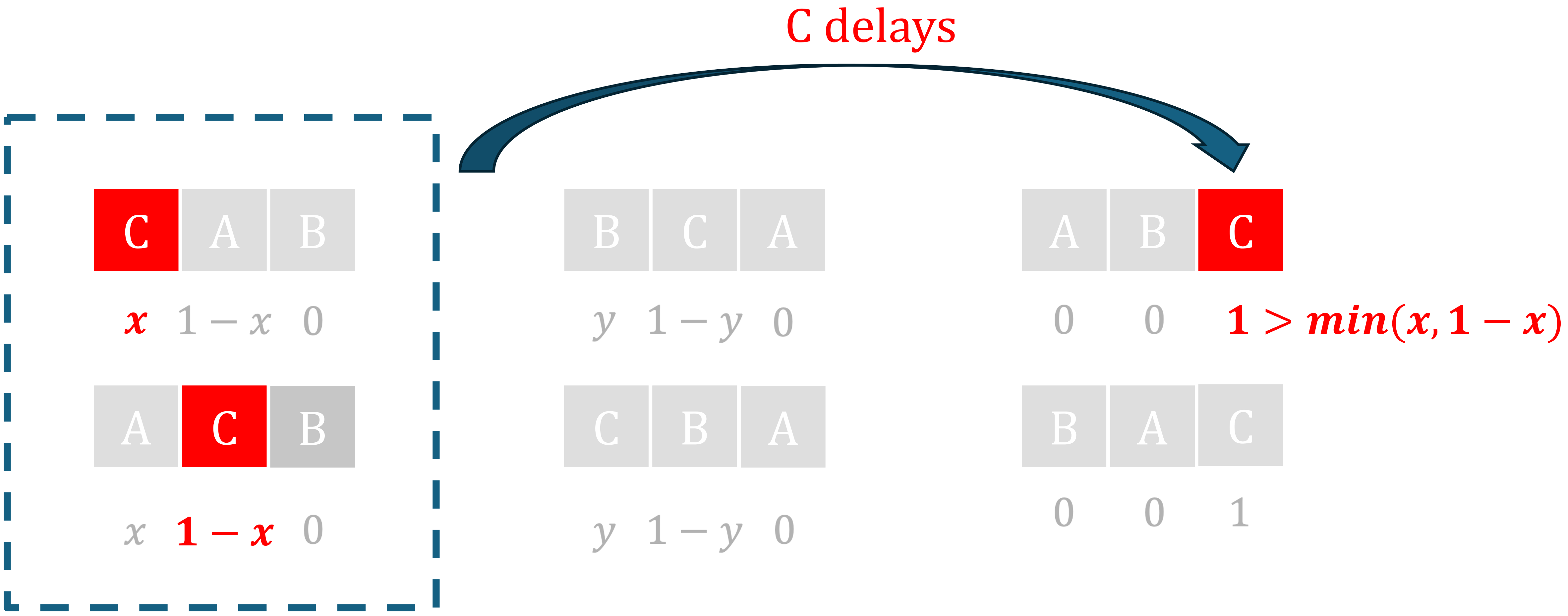
SF $\Rightarrow$ $SV_C = 2/3 = (x + 1 - x + y + 1 - y + 1 - \Delta_1 + 1 - \Delta_2)/6$

$\Rightarrow$ $\Delta_1 = \Delta_2 = 0$



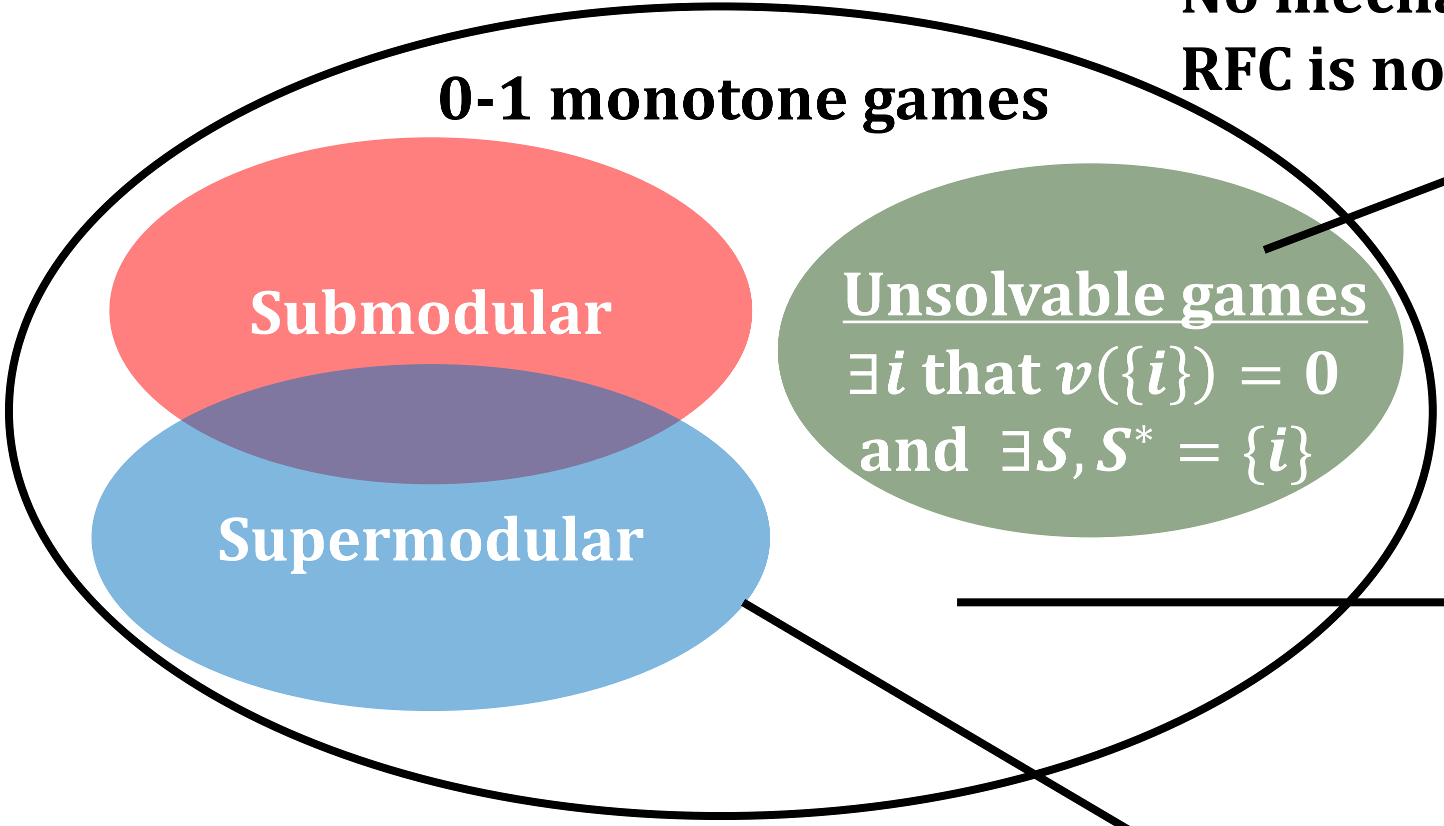
$(A \lor B) \land C$ is unsolvable

SF + OIR → not I4EA



Summary of Properties

Characterization of Unsolvable Games:
No mechanism satisfying OIR SF and I4EA.
RFC is not I4EA.



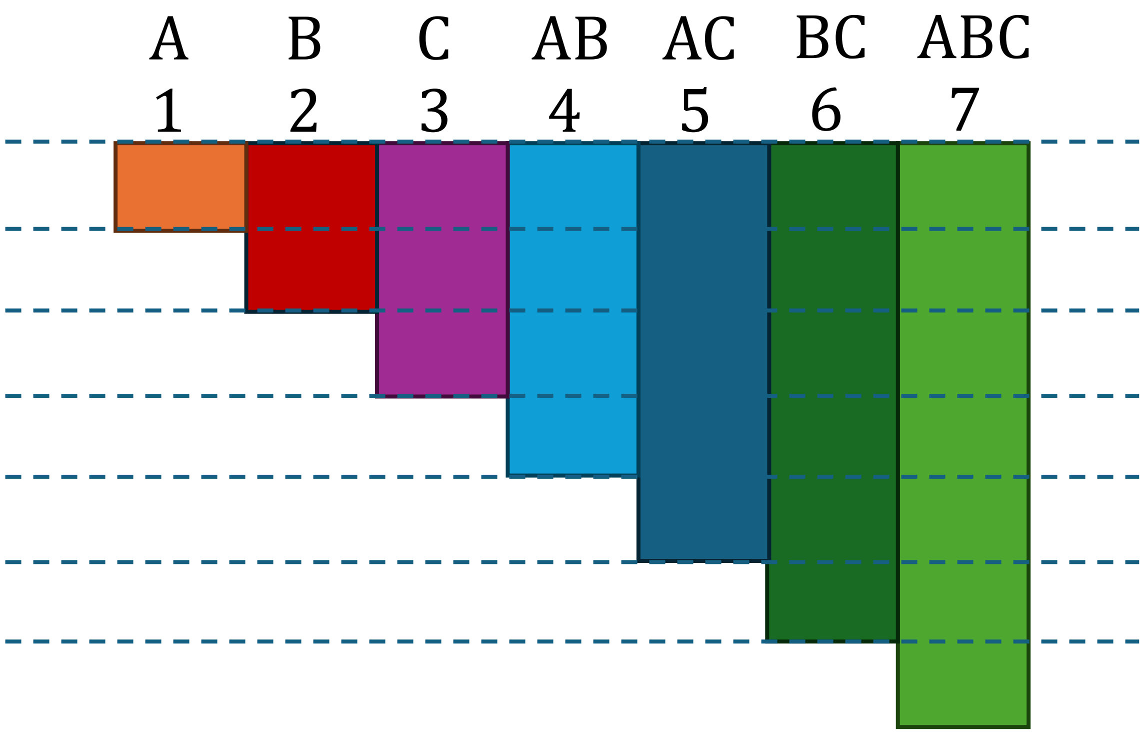
Completeness of RFC:
RFC is I4EA, SF and OIR.

submodular and supermodular games are solvable

Extend RFC to General Valued Monotone Games

Definition: eRFC

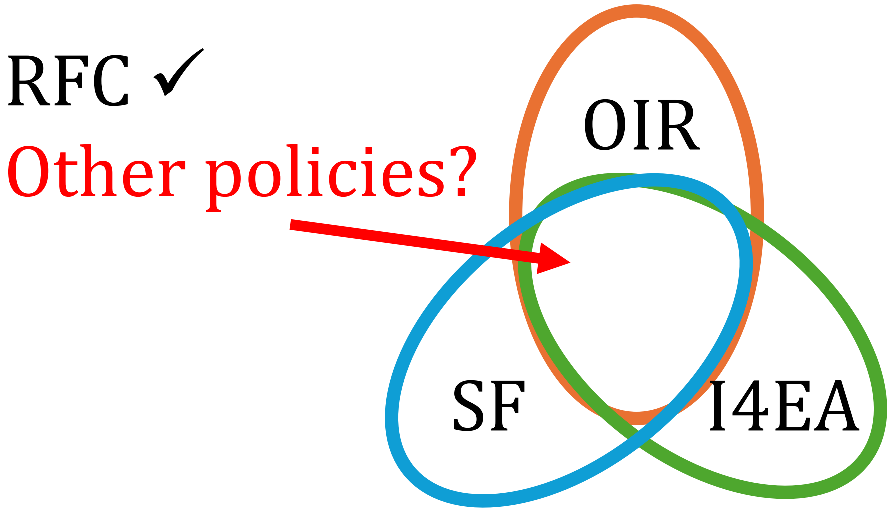
(1) decompose game online (2) accumulate the share



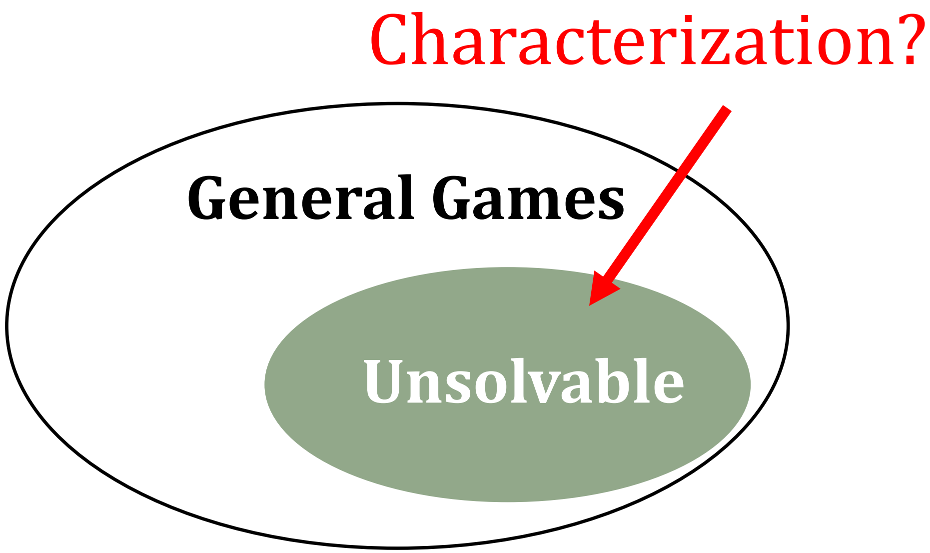
A	B	C	AB	AC	BC	ABC
1	2	3	4	5	6	7
1	1	1	1	1	1	1
	1	1	1	1	1	1
		1	1	1	1	1
			1	1	1	1
				1	1	1
					1	1
						1

0-1 valued components

Future Work



(1)



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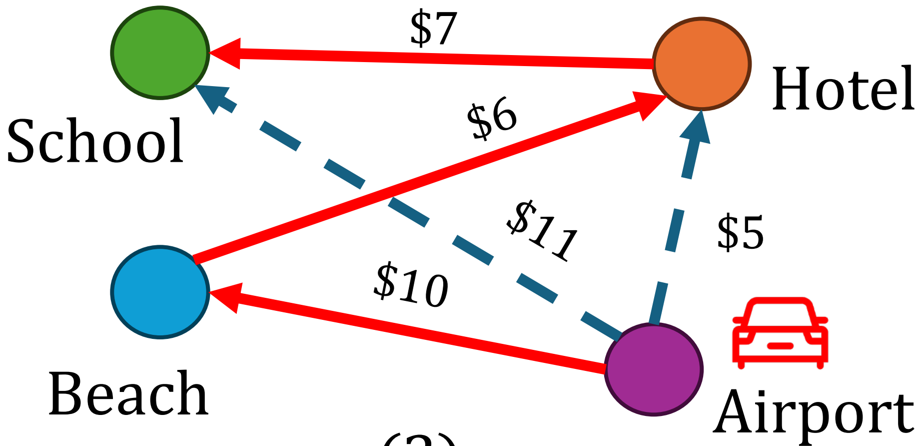

(3)