




A Simple Task Allocation with Execution Uncertainty

- **SETTING:** A broken car can be fixed in three different garages with different repair suggestions (different costs) and each garage may fail to satisfy the driver's need or simply couldn't fix the problem with their suggested repair, i.e., the execution uncertainty which is defined by **probability of success** $\in [0, 1]$.
- **QUESTION:** Which garage should the driver choose to "successfully" repair the car with a minimal cost?

Garage	Repair Suggestion (Cost)	Rating (<i>Probability of Success</i>)
	\$100	★ ★ ★ ☆ ☆ (0.5)
	\$150	★ ★ ★ ★ ☆ (0.8)
	\$200	★ ★ ★ ★ ★ (0.7)

- **ASSUMPTIONS:**

- Both **cost** and **probability of success** of each garage are their **private information**.
- **Cost is fixed** (independent of the *probability of success*).

- **CHALLENGE:** To successfully fix the car with a minimal cost, the driver need to **incentivise garage to reveal their true cost and true probability of success**.

- **SOLUTION:** Apply Vickrey-Clarke-Groves (VCG) mechanism? **NO!**

- Assume the driver's budget for the repair is 300 (i.e., valuation for a successful repair), the social welfare to repair in the garages are $300 \times 0.5 - 100 = 50$, $300 \times 0.8 - 150 = 90$ and $300 \times 0.7 - 200 = 10$ respectively.
- The driver will **choose the second garage** (which gives the highest expected social welfare) to repair his car and he will **pay**

$$300 \times 0.8 - 50$$

- **Problem:** the garage's payment depends on their *probability of success* (**violates truthfulness**)

A Good Solution: Fault Tolerant Mechanism

- **SOLUTION:** Post-Execution Verification Based Payment [Porter et al. 2008]

- **reward** the garage for successfully repairing the car,
- **charge** the garage for failing the task.

For the above example,

- the garage **receive** $300 - 50$ for successfully repairing the car,
- the garage **pay** 50 for failing the task.

Therefore the garage's **expected payment** is

$$- 0.8 \times (300 - 50) - (1 - 0.8) \times 50 = 300 \times 0.8 - 50$$

Question: Is the garage's expected payment different from the VCG payment?

- Yes, the garage can't change their probability of success 0.8 in the verification-based payment, while they can misreport under VCG as VCG uses their reported probability of success for the payment computation.

Our Contribution

We **generalise the task allocation with execution uncertainty**:

- n agents $N = \{1, \dots, n\}$.
- a finite task allocation space T , each $\tau \in T$ is defined by $\tau = (\tau_i)_{i \in N}$, where τ_i is a set of tasks assigned to agent i .
- $p_i^\tau \in [0, 1]$ is the probability that i will successfully complete her tasks τ_i . $p_i = (p_i^\tau)_{\tau \in T}$ is i 's **probability of success (PoS)** profile.
- i 's valuation is defined by $v_i : T \times [0, 1]^N \rightarrow \mathbb{R}$. v_i **considers costs, externalities, and task interdependences**.

And **characterise the applicability of the Post-Execution Verification (PEV)-based mechanism** (see the paper for other characterisations):

- PEV-based mechanism is **ex-post truthful** if and only if for all $i \in N$, v_i is multilinear in PoS (**risk-neutral**).
- PEV-based mechanism is **individually rational** if and only if each agent's valuation is non-negative if she is not allocated to do any task (i.e., she is not hurt by the others' executions when she does not do any task in any allocation).

The Key Literature

- [1] R. Porter, A. Ronen, Y. Shoham, and M. Tennenholtz. Fault tolerant mechanism design. *Artif. Intell.*, 172(15):1783–1799, Oct. 2008.
- [2] S. D. Ramchurn, C. Mezzetti, A. Giovannucci, J. A. Rodriguez-Aguilar, R. K. Dash, and N. R. Jennings. Trust-based mechanisms for robust and efficient task allocation in the presence of execution uncertainty. *J. Artif. Int. Res.*, 35(1):119–159, June 2009.
- [3] S. Stein, E. Gerding, A. Rogers, K. Larson, and N. Jennings. Algorithms and mechanisms for procuring services with uncertain durations using redundancy. *Artificial Intelligence*, 175(14-15):2021–2060, 2011.
- [4] V. Conitzer and A. Vidali. Mechanism design for scheduling with uncertain execution time. In *Proceedings of the Twenty-Eighth AAAI Conference on Artificial Intelligence*, pages 623–629. AAAI Press, 2014.