Incentives for Early Arrival

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Abstract

Incentives for Early Arrival (I4EA) is a novel concept for online cooperative games introduced in an award-winning paper by Ge *et al.* (2024). The aim of I4EA is to encourage players to join a collaboration as soon as they become aware of it, a new study with significant real-world applications, including data collection and venture capital finance. This paper provides an in-depth discussion of I4EA and highlights its importance across various domains.

Introduction

Collaboration is a fundamental and crucial aspect of human behavior that drives economic and social development. It has been extensively examined through cooperative game theory (Davis and Maschler 1965), which explores scenarios where players can form coalitions, cooperate, and make binding agreements. In cooperative games, players work together towards a shared goal, such as maximizing rewards (value sharing (Shehory and Kraus 1998)) or reducing costs (cost sharing (Jain and Mahdian 2007)).

To ensure effective collaboration among players in a coalition, it is essential to design a mechanism for allocating value/costs among them. This allocation mechanism, known as a solution concept, is typically grounded in various notions of fairness. Prominent examples of solution concepts include the Shapley value (Shapley 1953), the Harsanyi dividend (Harsanyi 1958), and the core (Von Neumann and Morgenstern 1947). For example, the Shapley value computes a player's value allocation based on their marginal contribution (the value increase) when they join a coalition. Since there is no fixed join order to decide their marginal contributions, Shapley simply considered all possible join orders (i.e., permutations) and averaged the marginal contributions across these orders to decide the player's value allocation. Shapley value is not necessarily in the core, which requires that no coalition (a subset of the players) has an incentive to leave the game.

In traditional cooperative game theory, a game is defined by specifying the value or cost for each coalition within a fixed and finite set of players N (known as the grand coalition). Solution concepts are typically based on the assumption that all players are already part of the game. This assumption overlooks the strategic process of coalition formation present in many real-world scenarios (Flammini et al. 2021; Bullinger and Romen 2023). For example, when investors decide to invest in a company, they may wait for others to invest first and choose the optimal time to join. Similarly, shops or restaurants considering joining a shopping mall do not necessarily do so collectively; optimizing the timing of their entry can be a key part of their strategy.

If the sequence and timing of coalition formation are strategic elements for players, solution concepts must be designed to incentivize desirable behaviors. A key incentive is to encourage players to join the coalition as soon as possible. This approach accelerates coalition formation and eliminates the strategic delay in deciding when to join. In practice, such an incentive helps start-ups secure necessary funding during their early, challenging stages and allows shopping malls to overcome the initial "cold start" period more effectively. This concept, known as the *incentive for early arrival* (I4EA), was first explored by Ge *et al.* (2024). Their research focused on online value sharing games where the players' identities and joining times are unknown before their arrival. They proposed a solution called *reward the first critical player* (RFC), which effectively encourages players to join as soon as they become aware of the game. Their work was recognized with a best paper award at AAMAS 2024.

In this paper, we will further explore and underscore the importance of studying incentives for early arrival beyond the context of value sharing games. We will examine its relevance and potential applications in various critical domains, including value sharing, cost sharing, financing, and marketing. Additionally, we will discuss how incentives for early arrival can be integrated with invitation incentives (Zhao 2021; Li et al. 2022). Invitation incentives, which encourage existing players to recruit new players via their social connections, represent a significant trend in mechanism design and are highly effective for expanding collaborations (Pickard et al. 2011; Zhang and Zhao 2022).

The Model

An online cooperative game is given by a triple (N, v, π), where N is the set of players who will join the game in the sequence defined by π, which is a permutation of N, and $v : 2^N \rightarrow \mathbf{R}$ is a set function which assigns a value (can be positive or negative) to each coalition $S \subseteq N$. Since the game is online, at any time point of the game, we do not know whether there will be new players joining and who will be the next joiner. That is, we do not have any prior information about N and π[1]. Of course, once a player or a set of players arrived, we know their value defined by v, which is a public knowledge as in the traditional cooperative games.

In a traditional offline cooperative game, all players are in the game simultaneously. Regardless of the joining order, the goal in both offline and online games is to determine how to allocate the total value among all players. In an offline game, this allocation is a one-time decision. However, in an online game, the allocation must be updated whenever a new player joins (simply because you cannot wait for more players, as you don't know if additional players will join. Otherwise, you could wait until everyone has arrived and treat it as an offline game). The mechanism used to determine these allocations is known as a *solution concept*. A well-known example is the Shapley value (Shapley 1953).

Definition 1 (Solution Concept) *A solution concept x of game (N, v, π) is defined by $(x_i(N, v, \pi))_{i \in N}$, where $x_i(N, v, \pi) \in \mathbf{R}$ represents the value allocation for player i.*

In the online game, the value allocation must be determined each time a new player joins, meaning that the final allocation for each player can depend on the order of arrivals. As a result, players may strategically choose when to join the game. For instance, a player who becomes aware of the game at time t might decide to delay joining until a later time t', if doing so results in a more favorable allocation for them. To discourage such strategic delay, the solution concept should ensure that waiting to join does not lead to a better outcome. This property is called *incentives for early arrival*. For example, if a start-up uses this type of solution concept to allocate shares among its shareholders, investors would be motivated to invest as soon as they are aware of the opportunity, rather than delaying their participation.

Definition 2 (I4EA) *We say a solution concept x is incentivizing for early arrival (I4EA) for game (N, v, π), if for all players $i \in N$, $x_i(N, v, \pi) \geq x_i(N, v, \pi')$, where the relative orders of $N \setminus \{i\}$ in both π and π' are the same, and i appears in π' no earlier than in π.*

Except for I4EA, there may be other desirable properties depending on the specific setting. For example, a player's total value allocation is non-decreasing as more players join, providing an incentive for them to stay in the game. We will discuss some of these important properties as we explore different settings later.

Value Sharing

Value sharing is a key focus of cooperative games, where the value each coalition generated is non-negative. If the value is non-decreasing when the coalition is getting larger, then we say the game is *monotonic*. If the game is monotonic, then the grand coalition (N) gives the highest value. In order to incentivize all players to collaborate in the grand coalition, we need to design a solution concept such that the players will not benefit from forming smaller coalitions, which is known as *core* (Von Neumann and Morgenstern 1947). Shapley value is another solution concept with many desirable properties, but it is not necessarily in the core (Shapley 1953).

In the online value sharing game, core and Shapley value cannot be computed before all players have arrived. If we simply recompute the value allocation whenever a new player arrives, then their value allocation may decrease with more players are joining, which will disincentivize them to stay. Except for the difficulty to compute the traditional solutions, they are also not suitable for incentivizing players to arrive early (I4EA). Therefore, Ge *et al.* (2024) for the first time proposed new solutions for I4EA.

Ge *et al.* proposed a new solution called reward first critical player (RFC) which satisfies I4EA for all monotonic 0-1 games that are possible to have I4EA. Under a 0-1 game, the value for each coalition is each 0 or 1. RFC simply allocates the marginal contribution of 1 (whoever creates) to first critical player (without whom the marginal value cannot be created). For a general value game, they also proposed a simple way to decompose the game into many 0-1 games and apply RFC to each of them and the sum of the value allocation for each player in the subgames is the final allocation to the player.

They also demonstrated that I4EA can be simply achieved by allocating all the value to the first arrival. This is certainly not fair, so they further proposed a fairness property called *Shapley-fair*, which requires that the averaged allocation of a player under all possible arrival sequences (which is the permutation of all players) is equal to his Shapley value. Of course, this is just one kind of fairness. In reality, there is just one arriving order, players also care the fairness of the value allocation under the specific order. Hence, we could also consider the distance between a player's allocation and his marginal contribution in each order. It looks also fair if a player's marginal contribution is high, his value allocation should be also high. It is clear this is not the case under RFC because the marginal contribution of a player might be completely allocated to another player.

There are also many other interesting problems that are not investigated yet. For example, under the general value games, what games can and cannot satisfy I4EA is unclear. What other solution concepts can we design for I4EA other than RFC? Although RFC is Shapley-fair, in reality, only one arrival sequence is realized. Would players actually consider RFC to be fair? We believe that conducting proper experiments with real participants will help us understand what is considered fair in practice and may lead to the discovery of other notions of fairness.

Financing

Financing is another important value sharing domain, which is a bit different from the value sharing game described

[1] One can extend the model to have some prior about N and π and utilize the prior in the solution concept.

above. For in equity finance or venture capital finance, venture capitalists invest a company in different rounds and at each round they share the value increase of the company in many different ways. Studies has shown that many businesses are unsuccessful in raising equity finance because they are not investment ready (Mason and Harrison 2004). Another important fact is that most venture capitalists do not want to invest a company too early and they seem to focus rather on commercialization of existing innovations and growth of the firm (Engel and Keilbach 2007). However, most start-ups really need strong support in the very early/immatural stage. Therefore, from a more healthy economic development point of view, venture capitalists should invest companies in a more early stage so that higher innovativeness of start-ups could happen.

However, the current financing market does not really incentivize investors to invest companies as early as possible. Also there is no fundamental theory on how the value increase during equity finance should be shared among the shareholders (Luo 2022). Hence, we believe it is very sensible to look at the theory to incentivize investors to invest companies as early as possible.

Different from the standard value sharing game, for each investor to invest a company, it comes with a cost (the money or resources offered to the company (Park et al. 2014)). Hence, the value allocation cannot be random. For example, we cannot simply take all the marginal value increase of one investor away (as what the RFC mechanism suggested to do). That is, we need more fairness constraints here and also value the resources each investor brings in. We cannot simply use the expected value allocation (like the Shapley-fairness propsed in (Ge et al. 2024)) to incentivize them to play the game.

Cost Sharing

Opposite to value sharing games, cost sharing games is another key portion of cooperative game theory. In a classical cost sharing game, a fixed group of players receive a service as a coalition, and the cost of the service is divided among the players. This game models many real-world applications such as electricity or water supply networks (Kar 2002; Gómez-Rúa and Vidal-Puga 2011; Trudeau and Vidal-Puga 2017). Compared to value sharing game, the key difference here is that players are motivated to receive a lower cost.

In the static games, value sharing games and cost sharing games share many solution concepts such as the core and Shapley value. However, in the online cost sharing games (Charikar et al. 2008; Furuhata et al. 2014; Zou, Dessouky, and Hu 2021), we cannot simply apply the solutions for online value sharing games. Especially, for I4EA, RFC (for online value sharing) is not applicable in the cost sharing game. The reason is that RFC may allocate the marginal contribution of a newly joined player to some players who have arrived earlier, which means in the cost sharing game, we need to reallocate the cost of an early arrived player to players who will arrive in the future. This process will dynamically change the role of some arrived players, which may eventually reallocate more cost to a early arrived player when more players are joining. Figure 1 shows an example of applying RFC under one 0-1 value sharing game and one 0-1 cost sharing game. If we simply apply RCF in the reserve order to decide the cost allocation in the cost sharing game, the cost of B increases when D arrives. This is a problem to keep players staying in the game (individual rationality issue).

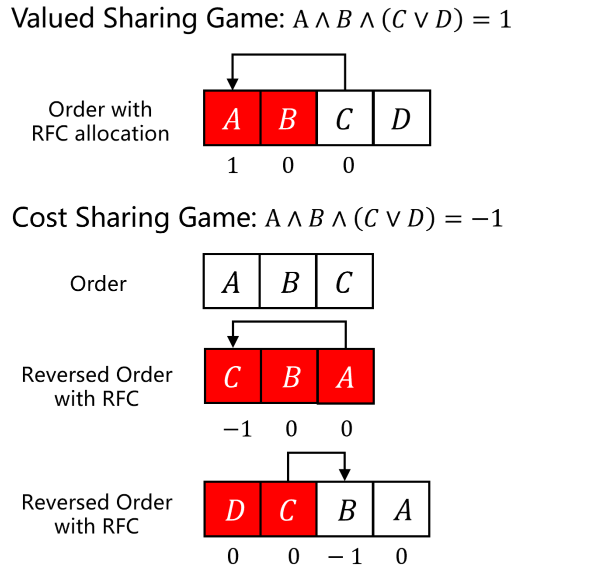

Figure 1: A running example of RFC, where the arriving order is A, B, C, D in both games.

Therefore, in the online cost sharing game, we need to design new solution concepts to satisfy both I4EA and individual rationality. The main challenge is that offering a value to an early arrived player is acceptable, but reallocating a cost to an early arrived player will disincentivize them to stay in the game (they will leave early, and destroy the coalition).

Marketing

Marketing is another important domain where firms have to strategically choose their strategies to make them competitive in different markets (Lieberman and Montgomery 1988). Numerous conceptual and empirical studies advance the notion that first-mover firms achieve long-term competitive advantages (Kerin, Varadarajan, and Peterson 1992; Varadarajan, Yadav, and Shankar 2008). Therefore, there exists certain incentive for firms to move first in marketing, but if we focus on a specific (smaller) market, this does not necessarily hold. For instance, for an newly opened shopping mall, the mall management team needs to strategically recruit shops and restaurants to join the mall. In this case, when there are only a few merchants in the mall, they are not able to attract enough customers and therefore, it is not beneficial for them to join the mall so early.

Therefore, it is also needed to investigate marketing mechanisms to incentivize firms to join a specific market such as a shopping mall as early as possible. A basic idea is that we should give benefits to the first-movers, but we should not do so with a very high cost. For example, we could offer space for free to attract firms in a mall, but it is not beneficial to the owner of the mall. One goal is that the mall could reallocated the rent to attract first-movers, but it should still get enough rent in the end, which in principle is what RFC does in the value sharing game (it is just a reallocation of the same value).

However, the marketing game is also very different from the standard cost sharing game mentioned above. For the shopping mall example, each player causes a cost (e.g. the rent) to join the game and at same time, they bring value/cost to one another in the game. A shop may benefit a restaurant but it may hurt another similar shop. Also a new joiner will get some business income which is not something that we can easily take away to redistribute. Hence, in this game, it is not just a reallocation of a fixed cost, there are very complex connections between players which play a vital role in the solution design.

Other interesting marketing games include new product trial programs (Robinson, Kalyanaram, and Urban 1994; Jensen 2003). On one hand, we want to incentive customers to try a new product as soon as possible, but on the other hand, the promotion should control the cost and make sure the promotion is beneficial in the long-run.

with Invitation Incentives

The incentives for early arrival will make sure that once a player is aware of the game, he/she will join immediately. However, how do the players know the game is another issue. One recent trend in mechanism design formally investigated the invitation incentives in various games including auction, matching and cooperative games (Zhao 2021; Li et al. 2022; Kawasaki et al. 2021; Zhang and Zhao 2022). In these studies, they proposed solutions to incentivize the existing players of a game to invite new players via their social connections (neighbours). Traditional solutions for these games do not have such incentives. For example, in a second price auction for auctioning one item (Vickrey 1961), a buyer is not incentivized to invite another buyer to compete with him/her. To solve this problem, we have to design proper reward mechanism to reward a player to invite others, but at same time, the mechanism does not want to run a deficit to do so.

Specifically, in a value sharing game, a player will compete with other players who have similar abilities in the game. For example, if we compute the Shapley value of a coalition, a player's Shapley value is decreased if we duplicate the player in the coalition. Therefore, the players are hesitated to invite new players. In order to incentivize them to invite new players no matter what abilities the new players have, we need to ensure that their value share is non-decreasing after inviting the others. One principle used to design such incentive is to share the marginal contribution of an invitee with his inviters (Zhang and Zhao 2022). This does not imply that joining the game earlier is beneficial because the invitation incentive only cares the connected players. If two players are not connected, the difficulty of designing I4EA is the same as we discussed in the previous sections. Therefore, how to design a solution to satisfy both properties is worth investigating.

Other Domains

Beyond the settings we have discussed, any online scenarios involving resource or task allocation might also benefit from incentives for early arrival:

- Crowdsourcing: For example, when crowdsourcing data to train large language models, we aim to gather the data as quickly as possible.
- Ticket Sales: For example, airlines often sell very cheap last-minute tickets, which can discourage early purchases. From the company's perspective, it may be beneficial to encourage customers to buy tickets as soon as they have a need.
- Technology Innovation: In some countries, innovation is more actively encouraged and protected than in others. To foster technological advancement, we need to create an environment that motivates and supports firms to take the lead in developing new technologies.
- Collaborative Machine Learning Model Training: In federated learning or large language model training, contributions from different parties–whether in terms of models or datasets–are essential. Evaluating these contributions is crucial to incentivize the contributors to join the training process as early as possible.

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