CS243: Introduction to Algorithmic Game Theory

Redistribution (Dengji ZHAO)

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Recap: The General Setting of Mechanism Design

- A set of n participants/players, denoted by N.
- A mechanism needs to choose some alternative from A (allocation space), and to decide a payment for each player.
- Each player $i \in N$ has a private valuation function $v_i : A \to \mathbb{R}$, let V_i denote all possible valuation functions for i.
- Let $v = (v_1, \cdots, v_n)$, $v_{-i} = (v_1, \cdots, v_{i-1}, v_{i+1}, \cdots, v_n)$.
- Let $V = V_1 \times \cdots \times V_n$, $V_{-i} = V_1 \times \cdots V_{i-1} \times V_{i+1} \times \cdots \times V_n$.

Recap: Myerson's Optimal Auction

- Given the bids $\mathbf{b}$ and the distribution of agents' valuations $\mathbf{F}$, compute virtual bids $b'_i = \phi_i(b_i) = b_i - \frac{1-F_i(b_i)}{f_i(b_i)}$.
- Run VCG on the virtual bids $\mathbf{b}'$ to get allocation $\mathbf{x}'$ and payment $\mathbf{p}'$.
- Output $\mathbf{x} = \mathbf{x}'$ and $\mathbf{p}$ with $p_i = \phi_i^{-1}(p'_i)$.

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Profit maximisation

Myerson's Optimal Auction maximises the seller's profit.

Recap: Auctions

- Truthful Mechanisms
  - Second-price auction
  - Generalization: VCG auctions
  - Optimal: Myerson's mechanism
- On social networks
  - Incentive diffusion mechanism (IDM)

Outline

Alternative Objective in Auctions

- Previously we focus on seller's revenue.
- What if the seller is not keen on revenue (e.g., an external agent or the government)?
- We now want to return the surplus to the agents.

Redistribution

Seeks to minimize net transfers from agents to an external body by return of VCG surplus to the agents.

Requirements

incentive compatibility Each agent will truthfully report her valuation v_i.

individual rationality Each agent will not suffer loss when she report her true valuation.

budget balanced The amount of extracted wealth that cannot be redistributed among the agents is 0.

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A First Attempt

Question

What if we uniformly return the VCG surplus, i.e., for each agent, we return v_2/n, where v_2 is the second highest bid among all agents?

Impossibility

Myerson-Satterthwaite Theorem

No mechanism is capable of achieving incentive compatibility, individual rationality, efficiency and budget balance at the same time.

Requirements

incentive compatibility Each agent will truthfully report her valuation v_i.

individual rationality Each agent will not suffer loss when she report her true valuation.

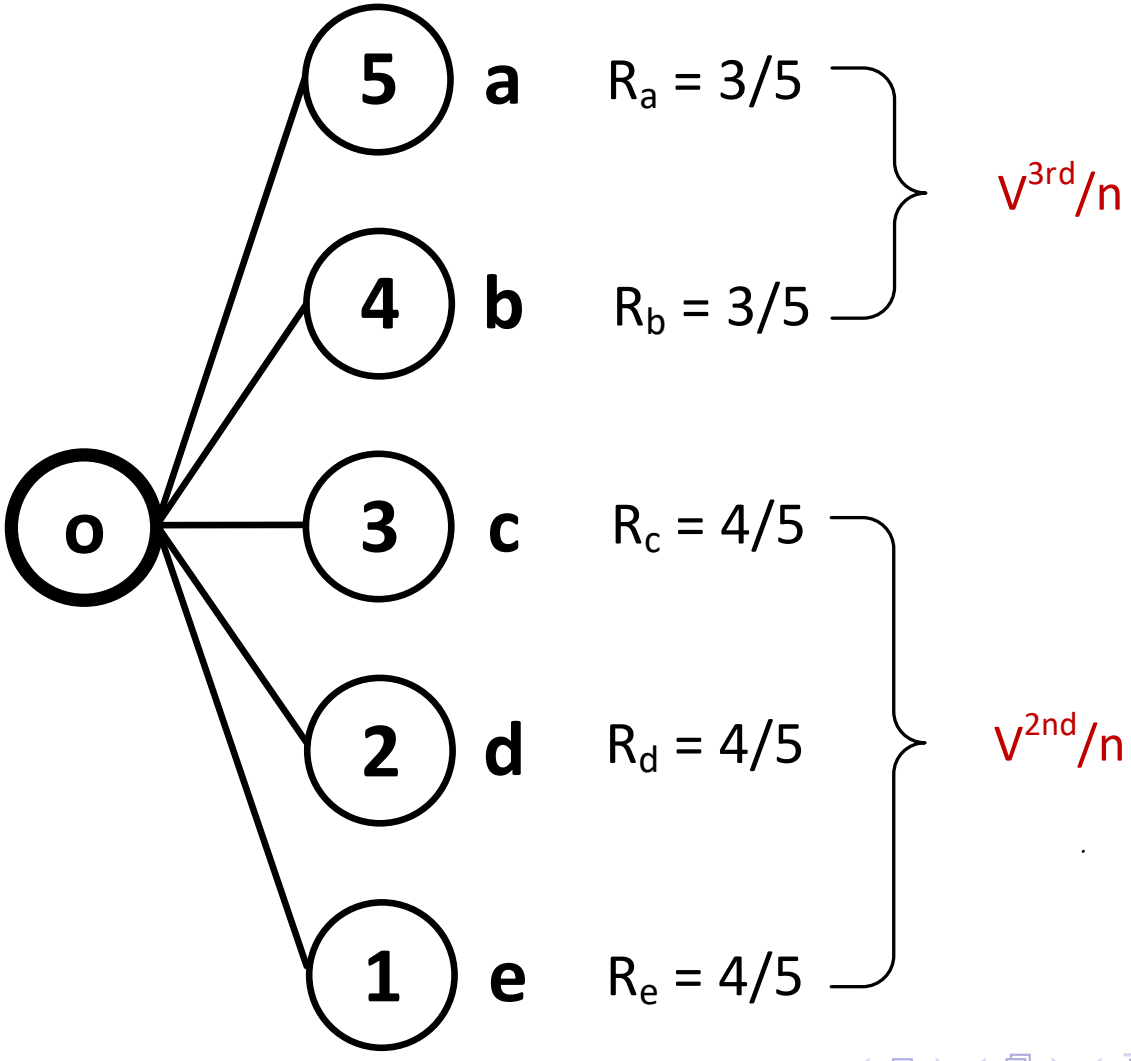
asymptotically budget balanced As the number of participating agents goes to infinity, the amount of extracted wealth that cannot be redistributed among the agents goes to 0.

Cavallo's Method

- Suppose agents $a_1, a_2, \ldots, a_n$ has bids $v'_1 \geq v'_2 \geq \cdots \geq v'_n$.
- Let a_1 be the winner and pays v'_2. (VCG)
- Return the surplus v'_2 back to agents as follows

$$r_i = \begin{cases} v'_3/n & \text{for } i = a_1, a_2 \\ v'_2/n & \text{for } i = a_3, \ldots, a_n \end{cases}$$

Cavallo's Method



Cavallo's Method

The amount not redistributed is

$$r_c = v_2' - \sum r_i = \frac{2}{n}\left(v_2' - v_3'\right)$$

Cavallo's Method

Theorem

Cavallo's Method is incentive compatible, individually rational, efficient and asymptotically budget balanced.

Advanced Reading

- *Optimal DecisionMaking With Minimal Waste: Strategyproof Redistribution of VCG Payments* by Ruggiero Cavallo (AAMAS 2006)

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Sponsored Search Auction (Dengji ZHAO)

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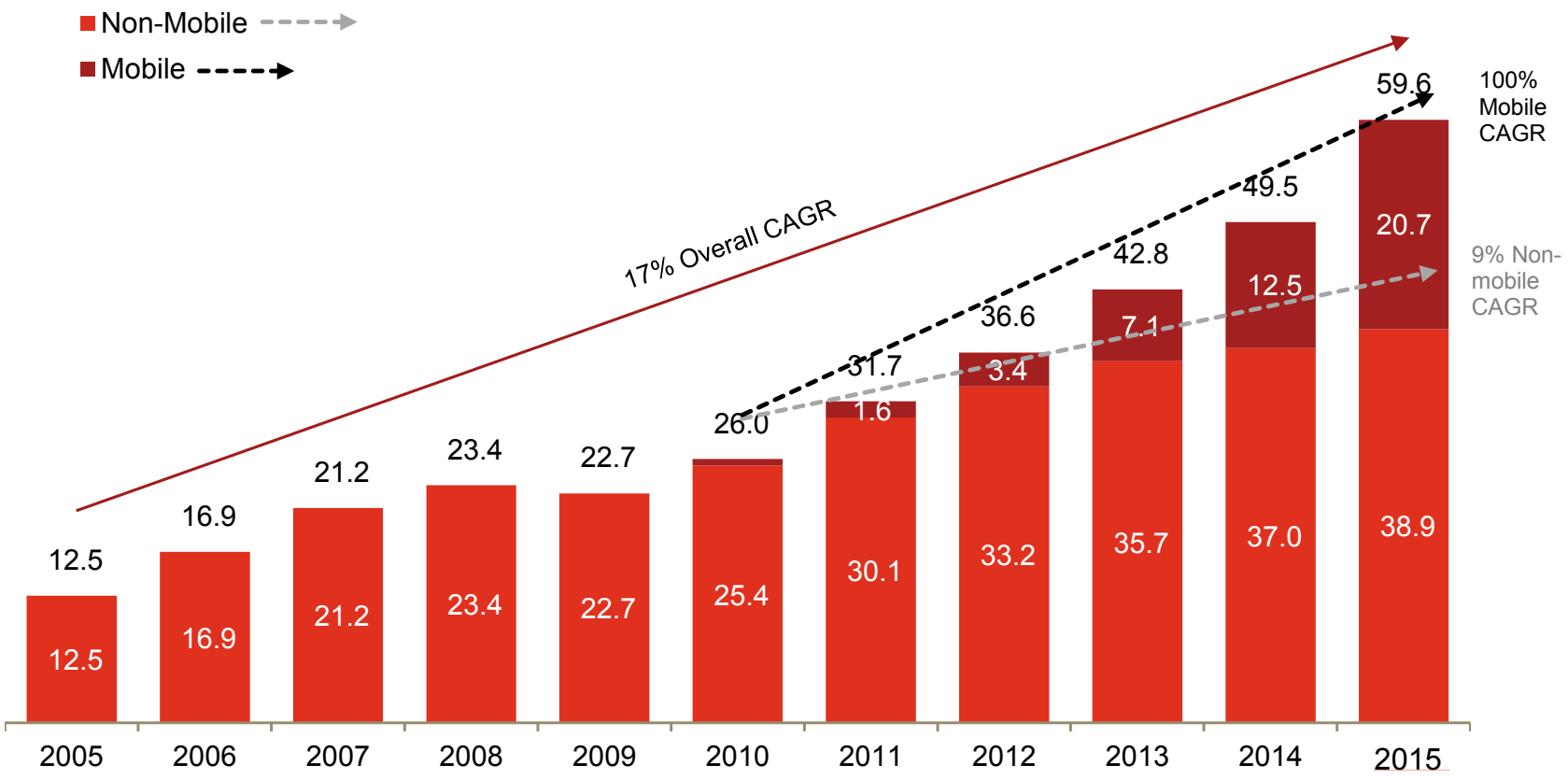
Sponsored Search Auctions

Sponsored Search Auction

- Used to sell ads slots by search engines such as Google, Baidu.
- Profit maximisation for the search engines?

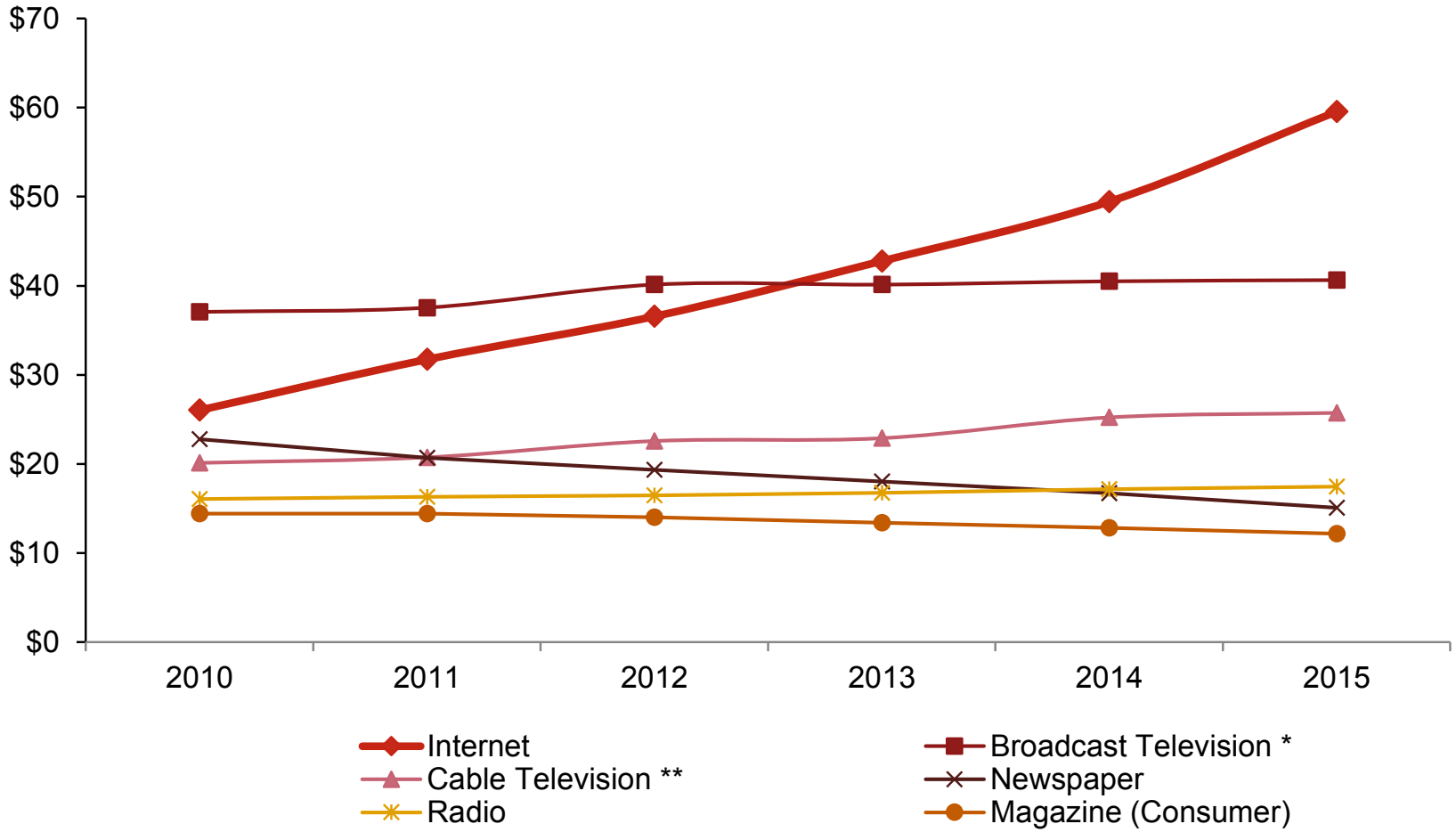
The Sponsored Search Market

Annual Revenue 2005-2015 ($ billions)



Source: IAB/PwC Internet Ad Revenue Report, FY 2015

The Sponsored Search Market

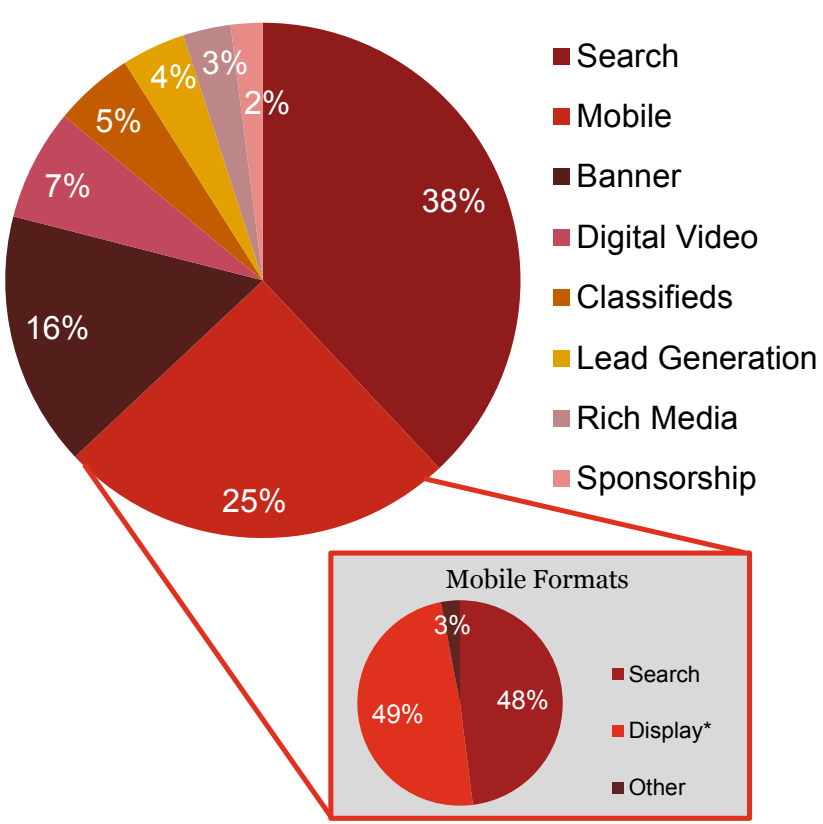


Sources: IAB/PwC Internet Ad Revenue Report, FY 2015; PwC

The Sponsored Search Market

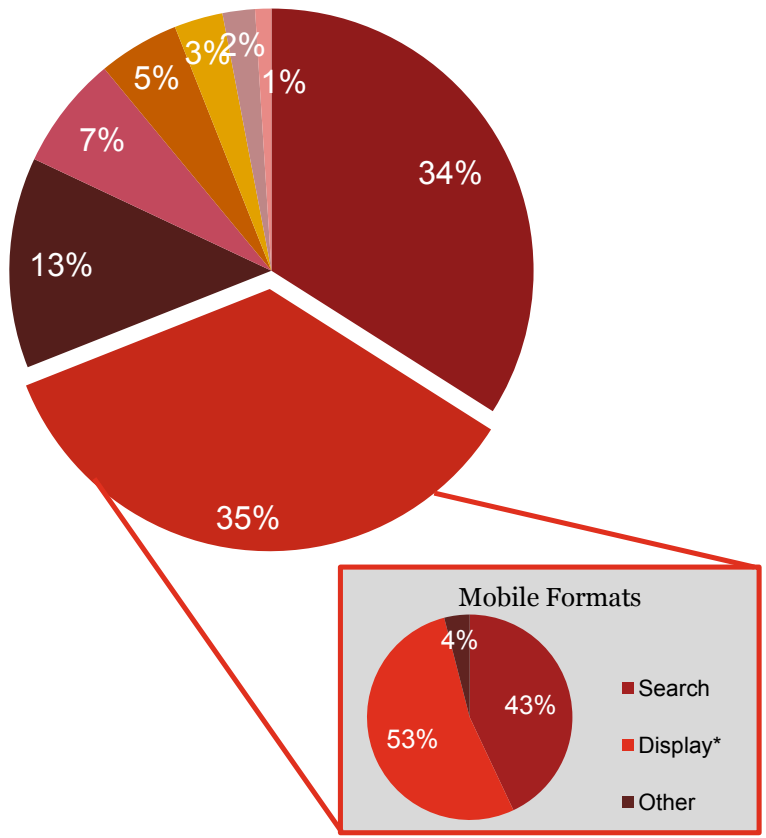
Ad formats – full year 2014

Total - $49.5 billion**



Ad formats – full year 2015

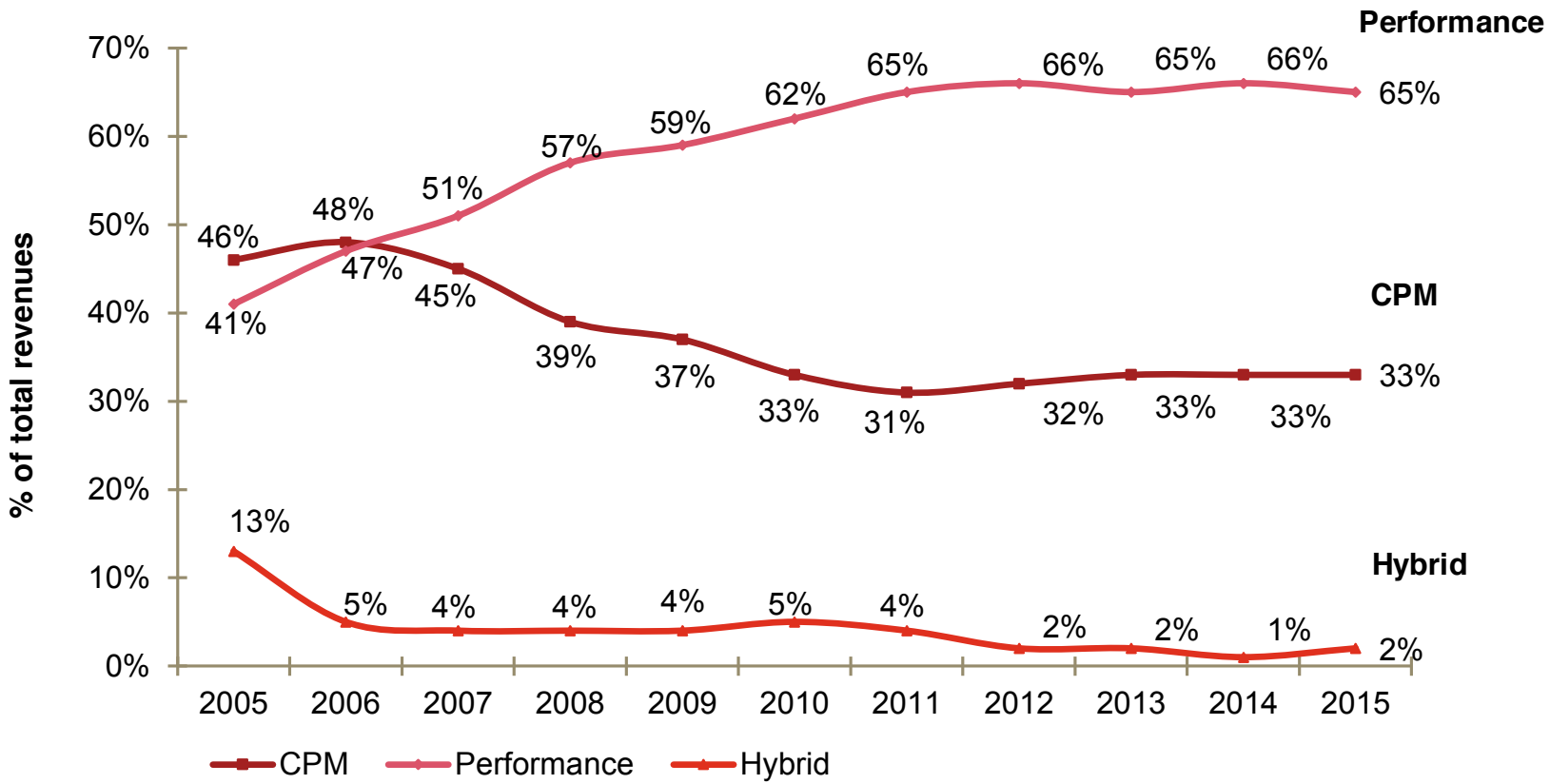
Total - $59.6 billion**



Source: IAB/PwC Internet Ad Revenue Report, FY 2015

The Sponsored Search Market

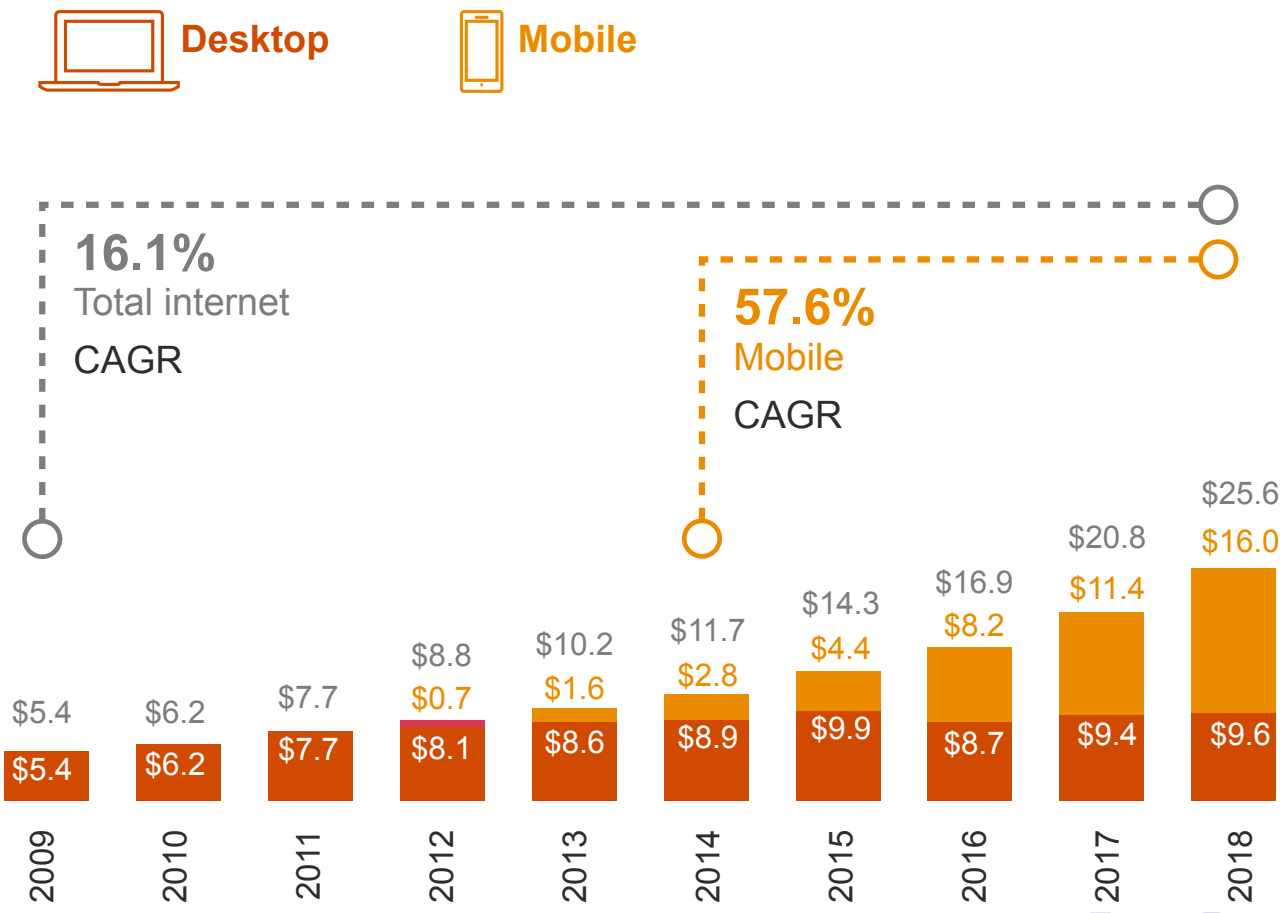
Internet ad revenues by pricing model*



Source: IAB/PwC Internet Ad Revenue Report, FY 2015

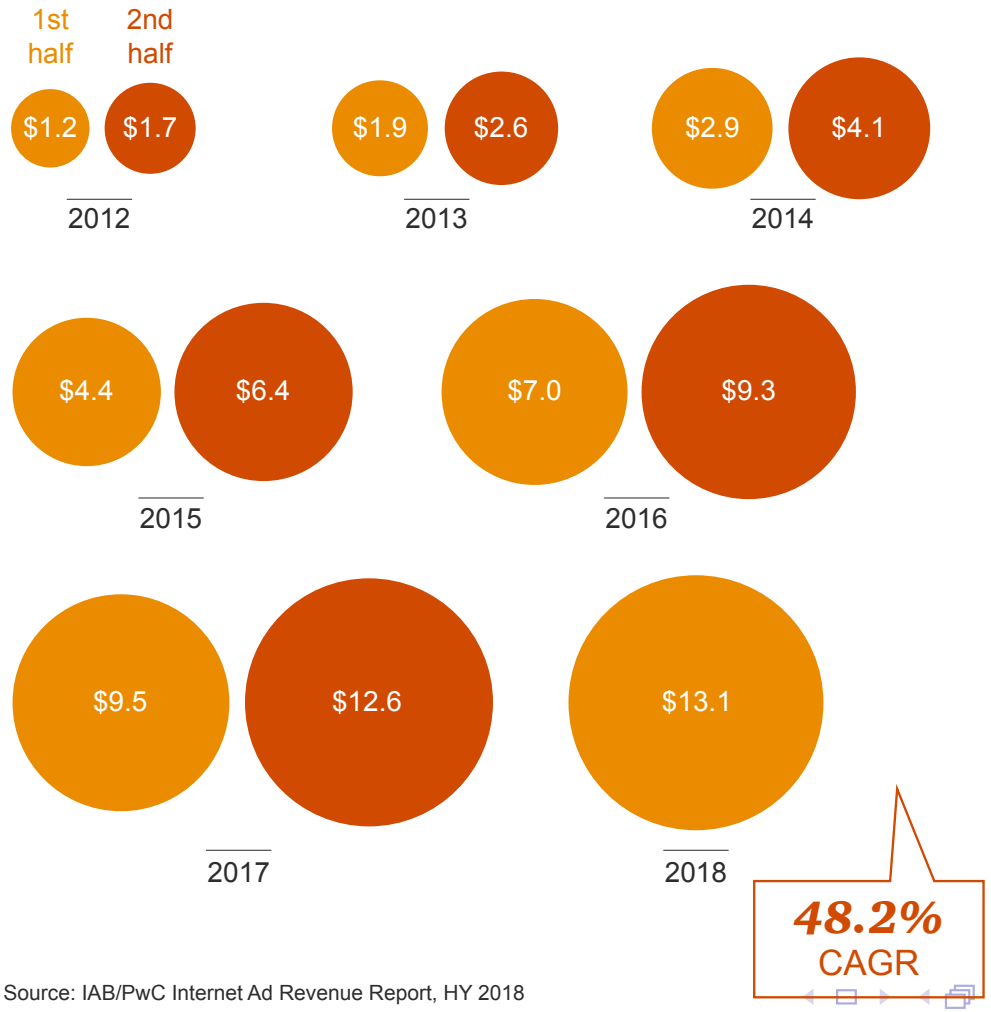
The Sponsored Search Market

Desktop vs. mobile internet advertising revenue
(Second quarter results, $ billions)



The Sponsored Search Market

Social media advertising revenue, half year results
($ billions)



Source: IAB/PwC Internet Ad Revenue Report, HY 2018

The Basic Model

- A set of advertisers/bidders (n), each specify a list of pairs of keywords and bids as well as a total budget (daily/weekly/monthly).
- A search engine with $m < n$ number of ad slots. The search engine estimate a click through rate α_{ij}, the probability that a user will click on the i-th slot when it is occupied by bidder j. Assume that $\alpha_{ij} \geq \alpha_{i+1j}$ for $i = 1, ..., m - 1$.
- The search engine also assigns a weight w_j to each advertiser j. The weight can be thought of as a relevance or quality metric.

Generalized Second Price (GSP) Auctions

For each search of a keyword, GSP does the following to allocate ads:

- Rank advertisers by their score $b_j w_j$.
- The highest score gets the first slot, the second highest score gets the second slot and so on.
- A bider pays per click the lowest bid necessary to retain his position.

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Two different variants:

1. Rank by bid (used by Overture): assume that $w_j = 1$
2. Rank by revenue (used by Google): assume that $w_j = \alpha_{1j}$

Efficiency in a Static Setting

- How to maximize social welfare?

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$$
\begin{aligned}
\max \quad & \sum_{i=1}^{k}\sum_{j=1}^{n} \alpha_{ij} v_j x_{ij} \\
\text{s.t.} \quad & \sum_{j=1}^{n} x_{ij} \leq 1 \qquad \forall i = 1, \ldots, k \\
& \sum_{i=1}^{k} x_{ij} \leq 1 \qquad \forall j = 1, \ldots, n \\
& x_{ij} \geq 0 \qquad \forall i = 1, \ldots, k, \ \forall j = 1, \ldots, n
\end{aligned}
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where $x_{ij} = 1$ if bidder j is assigned to slot i and zero otherwise.

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- What will be the payment under VCG?

The VCG Payments

- Consider three bidders $1, 2, 3$ with $v_1 > v_2 > v_3$ for one keyword and two slots.
- Suppose that $\alpha_{ij} = \mu_i$ with $\mu_1 > \mu_2$ (CTR are bidder independent).

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Revenue Maximization

- How to maximize search engine's revenue?

The Dynamic Setting

- What will happen if the game is repeated?

Advanced Reading

- AGT Chapter 28. Sponsored Search Auctions