Mechanism Design Powered by Social Interactions

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A tutorial @ AAMAS, IJCAI 2019

2009 DARPA Red Balloon Challenge

- The $40,000 challenge award would be granted to the first team to submit the locations of 10 moored, 8-foot, red weather balloons at 10 previously undisclosed fixed locations in the continental United States.

2009 DARPA Red Balloon Challenge

MIT Crowdsourced Solution (The Winner):

- "*We're giving $2000 per balloon to the first person to send us the correct coordinates, but that's not all – we're also giving $1000 to the person who invited them. Then we're giving $500 whoever invited the inviter, and $250 to whoever invited them, and so on ...*"
- got over 5,000 of participants, won the competition in under 9 hours.

2009 DARPA Red Balloon Challenge

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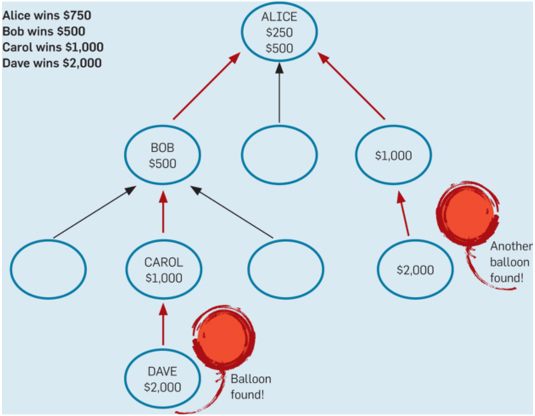


- Pickard, G., et al., Time-Critical Social Mobilization. Science, 2011. 334(6055): p. 509-12.

PinDuoDuo (like Groupon)

What are the incentives?

More participants, higher chance to win!!!

- 2009 DARPA Red Balloon Challenge
  - Inviting more friends has higher chance to win (higher utility)
- PinDuoDuo
  - Inviting more friends has higher chance to get cheap items (higher utility)

What if it is a competition?

- Resource allocation (auctions)
- Task allocation (crowdsourcing)
- Information propagation with budget
- Social choice (voting)

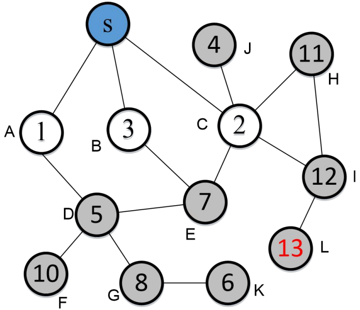
More participants means lower chance to win!!!

Diffusion Mechanism Design

Mechanism Design on Social Networks

Design mechanisms/markets under competitive environment such that participants are incentivized to invite more participants/competitors to join the mechanisms.

Starter: Promote a Sale via Social Networks



- The seller (blue node) sells one item and has only three connections/neighbours in the network (A,B,C).
- Each node is a potential buyer and the value is her highest willing payment to buy the item (valuation).
- The seller's revenue of applying second price auction (VCG) without promotion is 2.
- but the highest willing payment in the network is 13.

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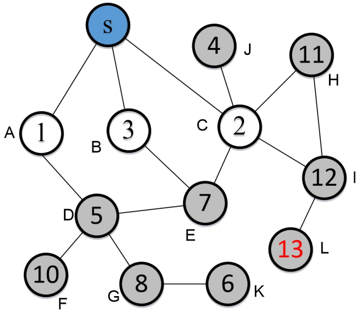

Question

How the seller could do to increase her profit?

Traditional Sale Promotions

Traditional sale promotions:

- Promotions via agents
- Keywords based ads via search engines such as Google
- Ads via social media such as WeChat, Facebook, Twitter

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Challenge

- The return of these promotions are unpredictable.
- The seller may LOSE from the promotions.

Tackle the Challenge

Build promotion inside the market mechanism such that

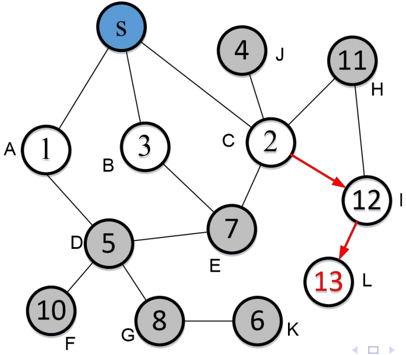
1. the promotion will never bring negative utility/revenue to the seller.
2. all buyers who are aware of the sale are incentivized to diffuse the sale information to all her neighbours.

"Diffusion Mechanism Design"

New Challenges

Why a buyer would bring more buyers to compete with her?

- only if their diffusion are rewarded, *but the seller doesn't want to lose*!
- we cannot just pay each node a fixed amount to incentivise them to diffuse the information.



Outline

1. Mechanism Design Review
   - The History
   - Second Price Auction (VCG)
2. Diffusion Mechanism Design

Outline

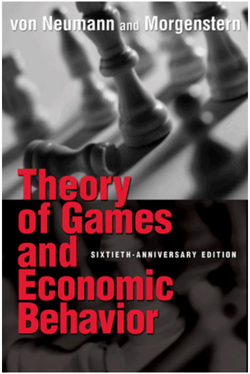
What is Mechanism Design

What is Mechanism/Market Design?

- it is known as Reverse Game Theory

What is Game Theory

- **Game theory** is the study of mathematical models of conflict and cooperation between intelligent rational decision-makers (wiki) [von Neumann and Morgenstern 1944].



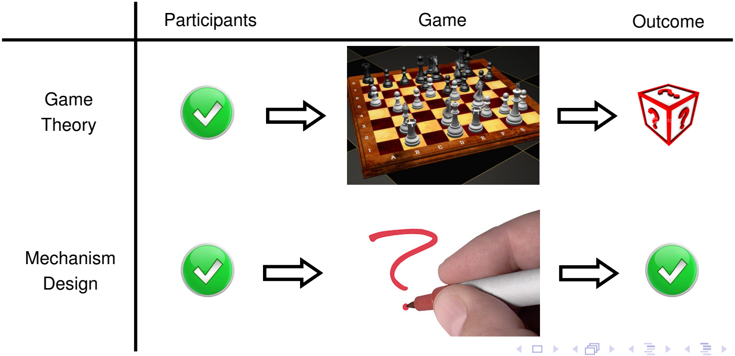
- Non-cooperative games: Go, poker, rock-paper-scissors
- Cooperative games: coordination games

Mechanism Design (Reverse Game Theory)

Mechanism Design is to answer...

Question

How to design a mechanism/game, toward desired objectives, in strategic settings?



Mechanism Design (Reverse Game Theory)

Mechanism Design is to answer...

Question

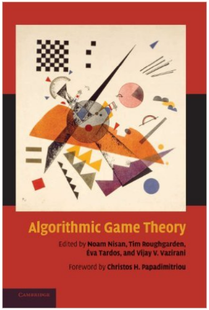
How to design a mechanism/game, toward desired objectives, in strategic settings?

Roger B. Myerson (born March 29, 1951, University of Chicago, US)

- Nobel Prize for economics (2007), for "having laid the foundations of mechanism design theory."
- *Eleven game-theorists have won the economics Nobel Prize*.

Algorithmic Game Theory (AGT)

- **Algorithmic game theory** is an area in the intersection of game theory and algorithm design, whose objective is to design algorithms in strategic environments (wiki) [Nisan et al. 2007].



- *Computing in Games*: algorithms for computing equilibria
- *Algorithmic Mechanism Design*: design games that have both good game-theoretical and algorithmic properties
- ...

Algorithmic Game Theory in Artificial Intelligence

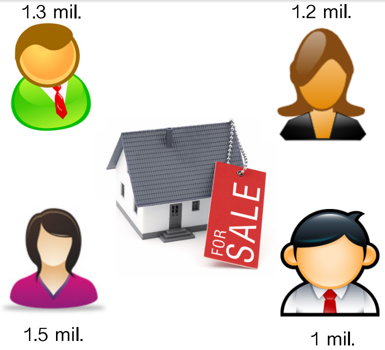
- Algorithmic game theory research in AI:
  - Game Playing: computation challenges, AlphaGo, poker
  - Social Choice: preferences aggregation, voting, prediction
  - Mechanism Design: the allocation of scarce resources, ad auctions
- Many IJCAI Computers and Thought Award (outstanding young scientists in artificial intelligence) winners had worked on AGT:
  - Sarit Kraus (1995), Nicholas Jennings (1999), Tuomas Sandholm (2003), Peter Stone (2007), Vincent Conitzer (2011), and Ariel Procaccia (2015)

Outline

A Mechanism Design Example

Design Goal

How can a house-seller sell her house with the "highest" revenue?



- **Challenge**: the seller doesn't know how much the buyers are willing to pay (their valuations).

A Mechanism Design Example

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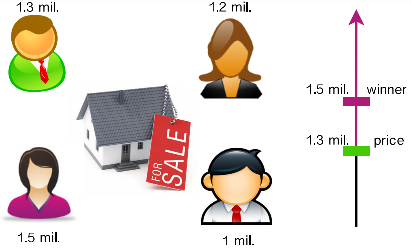

Solution: Second Price Auction (Vickrey Auction/VCG)

- Input: each buyer reports a price/bid to the seller
- Output: the seller decides
  - *allocation*: the agent with the highest price wins.
  - *payment*: the winner pays the second highest price.

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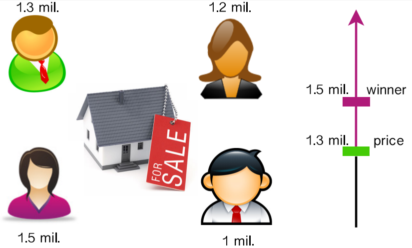

Solution: Second Price Auction (Vickrey Auction/VCG)

Properties:

- Efficient: maximising social welfare
- Truthful: buyers report their valuations truthfully

Is this the BEST the seller can do?

Question

What can the seller do to FURTHER increase her profit?

- estimate a good reserve price [Myerson 1981]
  - *requires a good estimation of buyers' valuations*
- promotions: let more people know/participate in the auction

Is this the BEST the seller can do?

Question

What can the seller do to FURTHER increase her profit?

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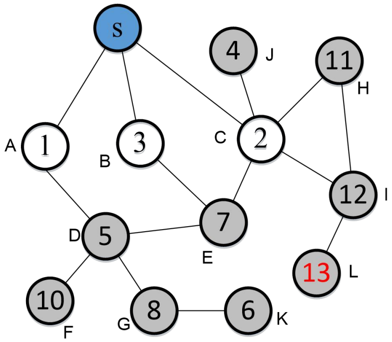
Recap: Promote a Sale via Social Networks



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   - Task Allocation
   - Information Propagation

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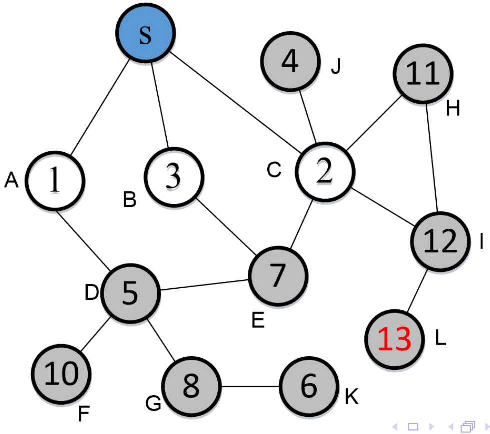
Our Solutions: Information Diffusion Mechanisms

- Bin Li, Dong Hao, Dengji Zhao, Tao Zhou: Mechanism Design in Social Networks. AAAI'17.
- Dengji Zhao, Bin Li, Junping Xu, Dong Hao, Nick Jennings: Selling Multiple Items via Social Networks. AAMAS'18.
- Bin Li, Dong Hao, Dengji Zhao, Tao Zhou: *Customer Sharing in Economic Networks with Costs*. IJCAI-ECAI'18.
- Bin Li, Dong Hao, Dengji Zhao, Makoto Yokoo: *Diffusion and Auction on Graphs*. IJCAI'19.
- Wen Zhang, Dengji Zhao, Hanyu Chen: *Redistribution Mechanism on Networks*. AAMAS'20.
- Wen Zhang, Dengji Zhao, Yao Zhang: *Incentivize Diffusion with Fair Rewards*. ECAI'20.
- Bin Li, Dong Hao, Dengji Zhao: Incentive-Compatible Diffusion Auctions. IJCAI'20.

The First Diffusion Mechanism

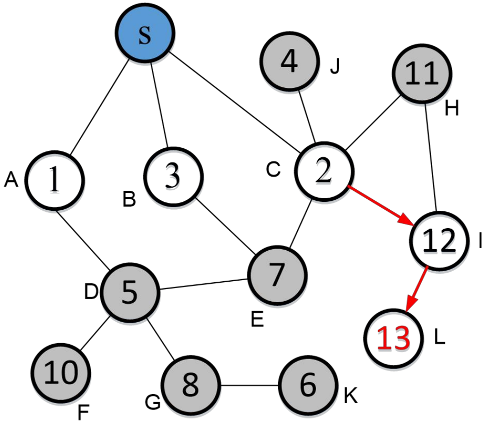
- Bin Li, Dong Hao, Dengji Zhao, Tao Zhou: **Mechanism Design in Social Networks.** AAAI'17.



Information Diffusion Paths

An information diffusion path from the seller to node L:

$s \rightarrow C \rightarrow I \rightarrow L$



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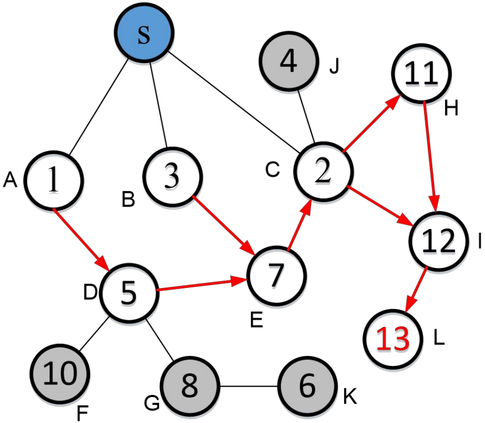
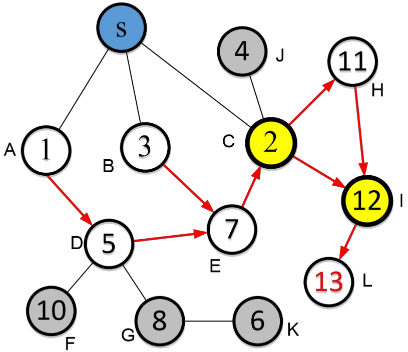


Diffusion Critical Nodes



Definition

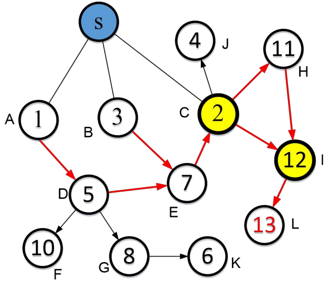
i is *j*'s diffusion critical node if all the information diffusion paths started from the seller *s* to *j* have to pass *i*.

- nodes C and I are L's only diffusion critical nodes.

Information Diffusion Mechanism [Li et al. AAAI'17]

The payment definition (second-price-like):

- If a buyer or one of her "*diffusion critical children*" gets the item, then the buyer pays the highest bid of the others (without the buyer's participation);
- otherwise, her payment is zero.



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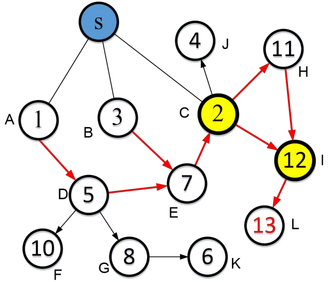


If the item is allocated to *L*, the payments of C, I and L are 10, 11, 12 respectively .

Information Diffusion Mechanism

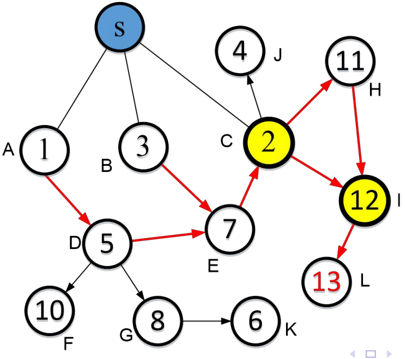
The allocation definition:

- Identify the node i with the highest bid and the node's diffusion critical node path $P_{c_i} = (c_i^1, c_i^2, ..., i)$.
- Give the item to the first node of P_{c_i}, the node pays to the seller and then decides to whether keep the item or pass it to the next node in P_{c_i}:
  - If the payment of the next node is greater than the bid of the current node, passes it to the next node and receives the payment from the next node; the next node makes a similar decision;
  - otherwise, keep the item.

The Information Diffusion Mechanism

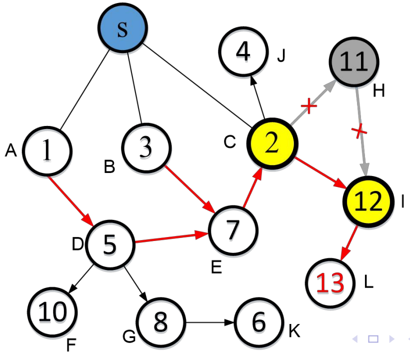
The outcome of the Information Diffusion Mechanism:

- the item is allocated to node I.
- node I pays 11 to C, C pays 10 to the seller.
- the utilities of I, C, the seller are 1, 1, 10.

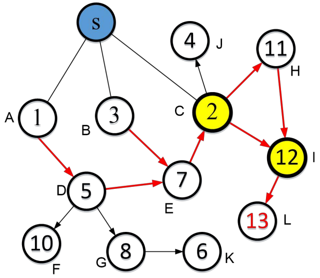


Why Buyers are Happy to Diffuse the Information?

- buyers receive the information earlier have higher priority to win the item (*C* chooses before *I* and *I* chooses before *L*).
- diffuse the information to more buyers will potentially increase their reward (if *C* does not invite *H*, her utility is 0).



Properties of the Information Diffusion Mechanism



- Truthful: report true valuation and diffuse the sale information to all her neighbours is a dominate strategy.
- Individually Rational: no buyer will receive a negative utility to join the mechanism.
- Seller's Revenue Improved: the seller's revenue is non-negative and is $\geq$ that of the VCG without diffusion.

What Next?

- Diffusion mechanisms for combinatorial exchanges
- Diffusion with costs and delays
- Network structure based revenue analysis
- Applications/implementations in the existing social networks
- Other mechanisms to further improve the revenue and/or the efficiency

Diffusion Mechanisms for Combinatorial Exchanges

Challenge

How to generalise the mechanism to combinatorial settings?

Diffusion Mechanisms for Combinatorial Exchanges

Consider the following simple setting:

- A seller sells three units of one commodity, e.g. MacBook computers.
- Each buyer has a diminishing marginal utility for consuming the goods.

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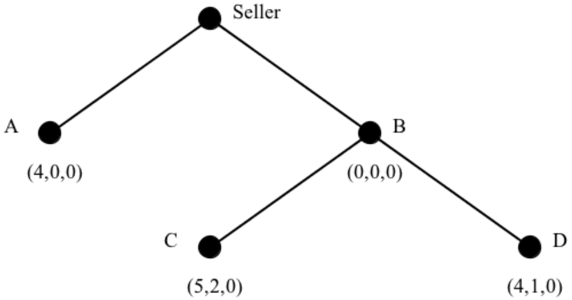
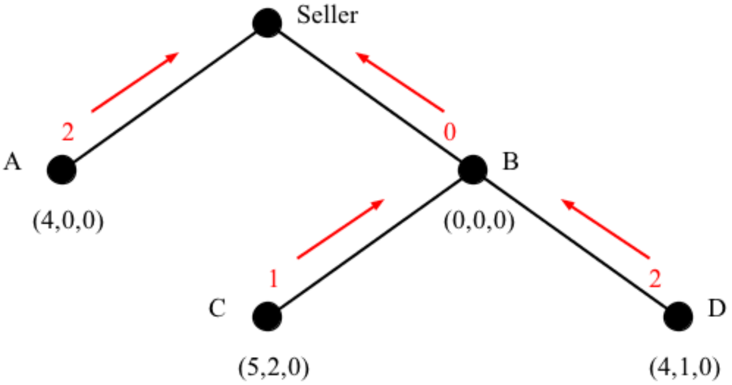


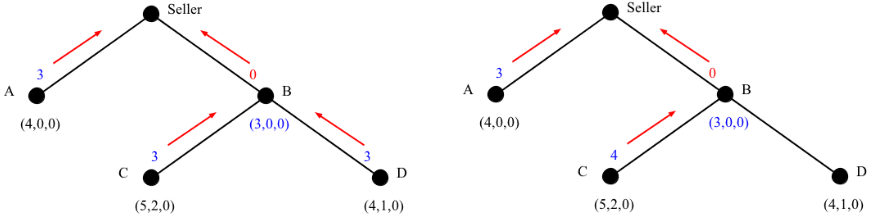
Diffusion Mechanisms for Combinatorial Exchanges

If we simply apply our information diffusion mechanism:



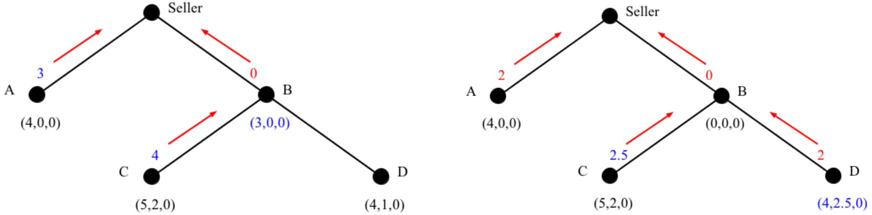
Diffusion Mechanisms for Combinatorial Exchanges

What if buyer B's valuation is $(3, 0, 0)$?



Diffusion Mechanisms for Combinatorial Exchanges

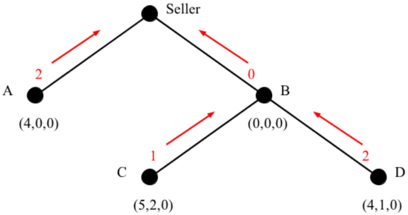
What if buyer D's valuation is $(4, 2.5, 0)$?

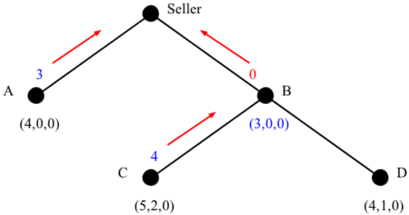


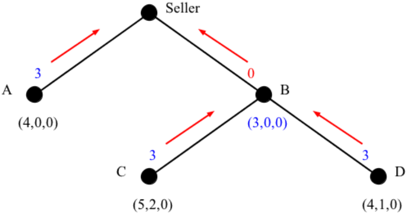
Diffusion Mechanisms for Combinatorial Exchanges

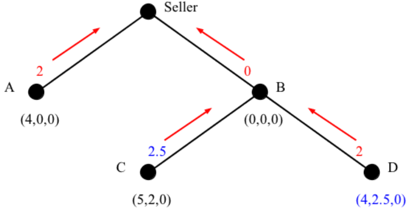
Challenge

There is a very complex **Decision Making** at each node!!!









Why is it so complex when there are multiple items?

To achieve truthfulness:

- The mechanism has to maximise each node's utility under truthful reporting.
- Each node's payment should not depend on her valuation.

The complexity we had:

- A node can influence her payments by controlling the items passed to her children.
- A node can influence the payments of her peers, without changing her own allocation and payments.
- This leads to a decision loop (very complex optimization) and may not able to maximise everyone's utility.

Reduce the Complexity

The Main Idea

A node CANNOT influence the payments she receives by controlling the items passed to her children.

Simplify the decision complexity we had:

- ~~A node can influence her received payments by controlling the items passed to her children.~~
- A node can influence the payments of her peers, without changing her own allocation and payments.
- ~~This leads to a decision loop and may not able to maximise everyone's utility.~~

One Solution: Sell Multiple Homogeneous Items

Selling Multiple Items via Social Networks [Zhao et al. AAMAS'18]

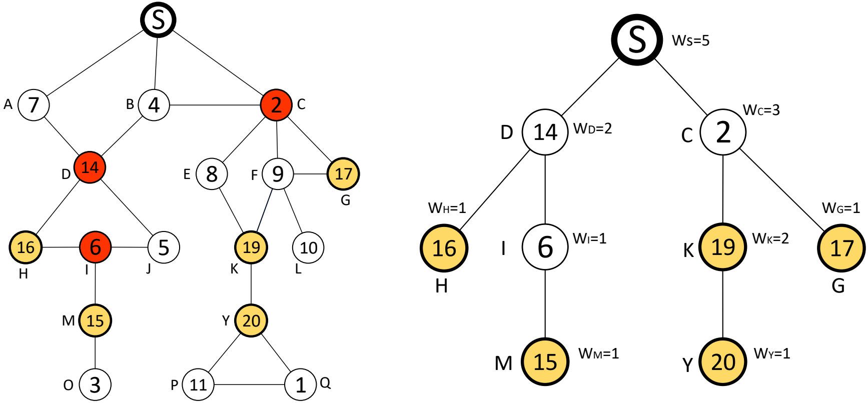
- generalises the result from [Li et al. 2017];
- agent *i*'s reward/payment doesn't depends on how many of *i*'s children received items;
- agent pays to the seller directly rather than to their parent;

The setting:

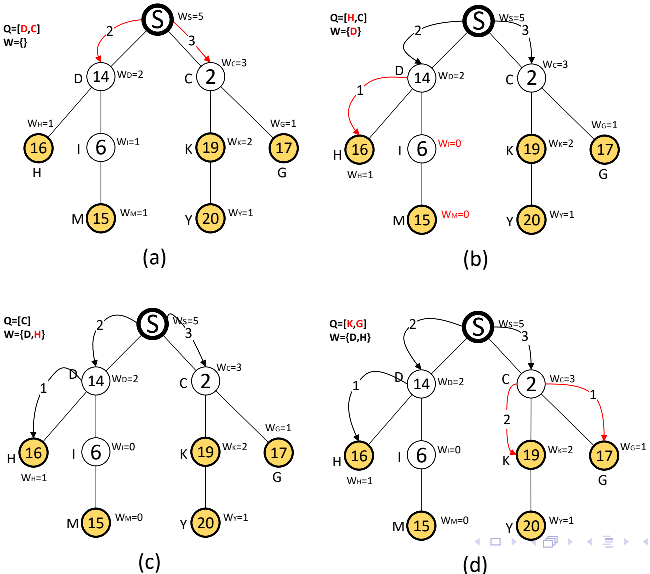
- A seller sells $\mathcal{K} \geq 1$ homogeneous items;
- each buyer requires at most one item (single-unit demand);
- the rest is the same as [Li et al. 2017].

The Generalised Diffusion Mechanism

Consider $\mathcal{K} = 5$:



The Generalised Diffusion Mechanism



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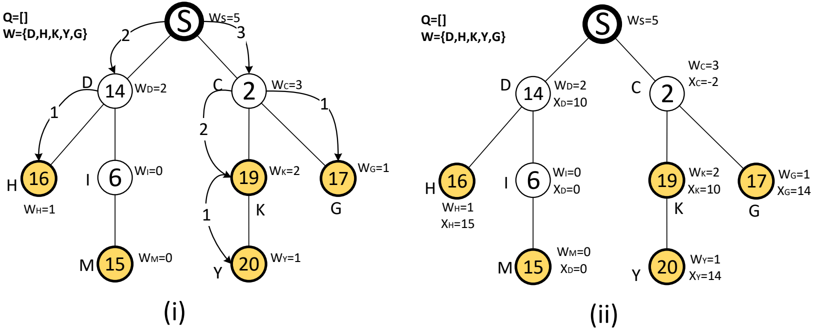


The Allocation Policy of the Generalisation

Node/buyer i receives one item if and only if

1. the top $\mathcal{K}$-highest valued children of i (and their parents, who are also i's children) do not participate
2. and i wins under the efficient allocation with their absence

given that all i's (critical) parents' allocation is determined and fixed.

The Payment Policy of the Generalisation

Node i's utility is the social welfare difference of the efficient allocation between

1. the top $\mathcal{K}$-highest valued children of i (and their parents, who are also i's children) do not participate (guarantees that i's payment does not depend on how many items i's children get)
2. and i (and all her children) does not participate

Formally, i's payment is:

$$\begin{cases} \mathcal{SW}_{-D_i} - (\mathcal{SW}_{-\mathcal{C}_i^{\mathcal{K}}} - v_i') & \text{if } i \in W, \\ \mathcal{SW}_{-D_i} - \mathcal{SW}_{-\mathcal{C}_i^{\mathcal{K}}} & \text{if } i \in \bigcup_{j \in W} \mathcal{P}_j(\theta') \setminus W, \\ 0 & \text{otherwise.} \end{cases}$$

where W is the set of nodes each of whom received one item.

Properties of the Generalisation

- Truthful: report true valuation and diffuse the sale information to all her neighbours is a dominate strategy for each node.
- Individually Rational: no node will receive a negative utility to join the mechanism.
- Seller's Revenue Improved: the seller's revenue is non-negative and is $\geq$ that of the VCG without diffusion.

Truthfulness and IR

Given i's payment:

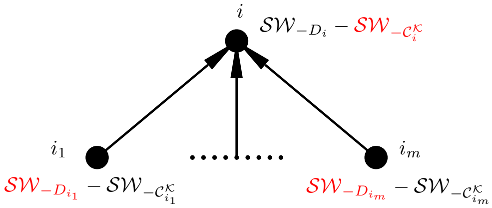
$$\begin{cases} \mathcal{SW}_{-D_i} - (\mathcal{SW}_{-\mathcal{C}_i^{\mathcal{K}}} - v_i') & \text{if } i \in W, \\ \mathcal{SW}_{-D_i} - \mathcal{SW}_{-\mathcal{C}_i^{\mathcal{K}}} & \text{if } i \in \bigcup_{j \in W} \mathcal{P}_j(\theta') \setminus W, \\ 0 & \text{otherwise.} \end{cases}$$

if i reports truthfully, i's utility is:

$$\mathcal{SW}_{-\mathcal{C}_i^{\mathcal{K}}} - \mathcal{SW}_{-D_i}$$

- $\mathcal{SW}_{-D_i}$ is the optimal social welfare without i's participation
- $\mathcal{SW}_{-\mathcal{C}_i^{\mathcal{K}}}$ is the optimal social welfare when the top $\mathcal{K}$-highest valued children of i (and their parents, who are also i's children) do not participate

Guaranteed Revenue Improvement for the Seller



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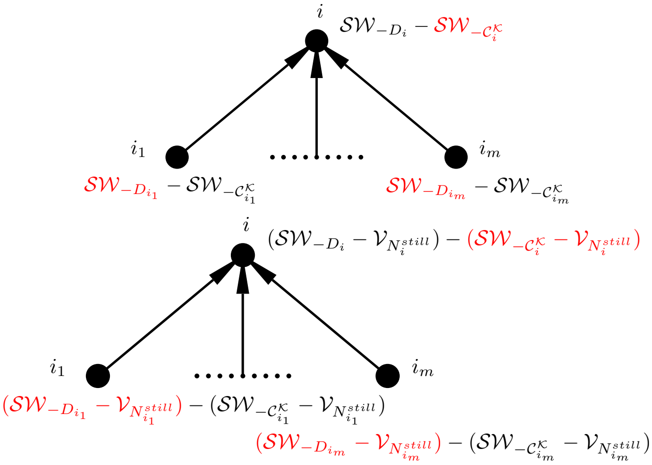

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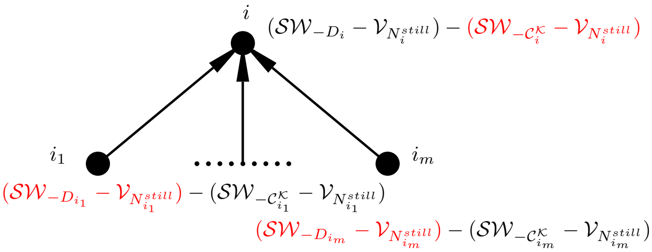
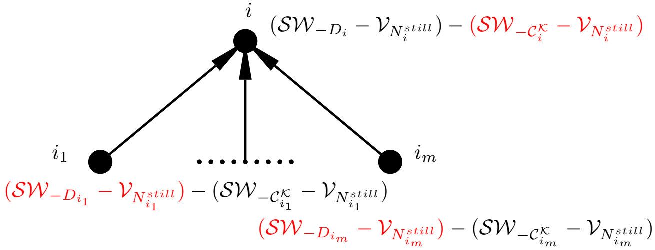


$$\mathcal{SW}_{-\mathcal{C}_i^{\mathcal{K}}} - \mathcal{V}_{N_i^{still}} \leq \sum_{i_l} (\mathcal{SW}_{-D_{i_l}} - \mathcal{V}_{N_{i_l}^{still}})$$

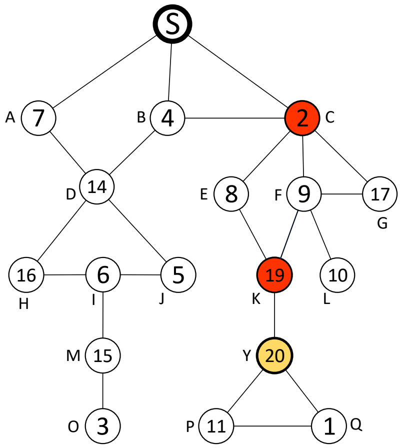
Guaranteed Revenue Improvement for the Seller



Theorem (Zhao et al. 2018)

The revenue of the generalised information diffusion mechanism is greater than or equal to $\mathcal{K} \times v_{\mathcal{K}+1}$, where $v_{\mathcal{K}+1}$ is the $(\mathcal{K}+1)$-th largest valuation report among r_s, assume that $|r_s| > \mathcal{K}$.

What happens when $\mathcal{K} = 1$?



Are the mechanisms fair?

- According to the theorem of small-world networks, the chance for a node to be a cut-point in a well-connected network is very low.
- We hope to give rewards to all the related buyers not only the cut-points on the paths to the winner.

Solution: Redistribute Rewards among Agents

Incentivize Diffusion with Fair Rewards [Zhang et al. ECAI'20]

- redistribute rewards among critical ancestors based on IDM
- all critical ancestors have positive expected utilities
- seller's revenue is not reduced

What if for non-profit purpose?

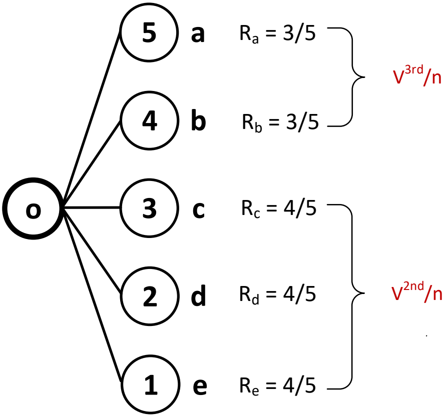
Redistribution mechanism is:

- to do the efficient resource allocation
- not for profit

Challenges:

- how to achieve a more efficient allocation?
- how to maintain wealth among agents?

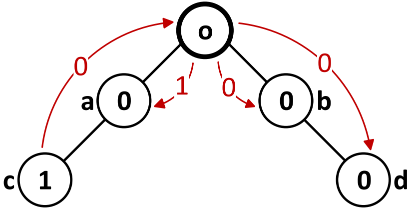
Cavallo Mechanism

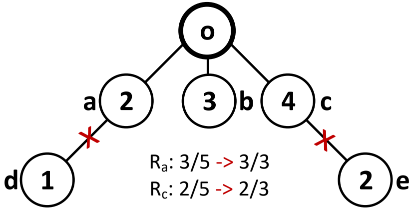


- **Allocation**: agent with the highest valuation wins the item
- **Payment**: winner pays the second price
- **Redistribution**: each agent receives the owner's revenue without her participation divided by the total number of agents

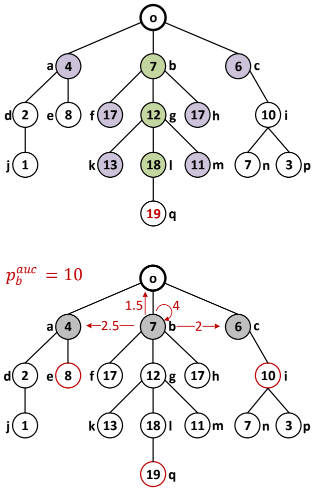
Why not Cavallo Mechanism on Networks?

- Run a deficit
- Disincentivize the diffusion





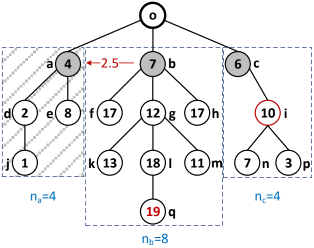
Redistribution Mechanism in Trees [Zhang et al. AAMAS'20]

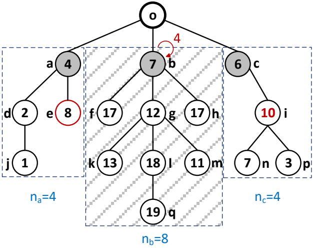


For each ancestor:

- **Allocation**: keep the item if her valuation is greater than or equal to her payment
- **Payment**: the highest valuation without her participation
- **Redistribution**: a monotone increasing function to the number of descendants

Details for Redistribution





For agent *a*:

- Without her participation, *b*'s payment becomes $v_i = 10$
- $R_a = 10 * \frac{4}{4+8+4} = 2.5$

For agent *b*:

- Without her participation, *c* will be the new ancestor and her payment is $v_e = 8$
- $R_b = 8 * \frac{8}{4+8+4} = 4$

Properties of the Mechanism

The mechanism works for all graphs only by updating the definition of critical ancestors.

- Individually Rational: each agent will not have a negative utility as long as she reports her true valuation.
- Truthful: reporting true valuation and inviting all her neighbours is a dominate strategy.
- Asymptotically Budget-balanced: when the number of participants goes to infinity, almost all the money will be redistributed back to the participants.
- No Deficit: the resource owner will never pay some extra money for the allocation.

A General Characterization of Diffusion Auction Mechanisms

Incentive-Compatible Diffusion Auctions [Li et al. IJCAI'20]

- characterize a sufficient and necessary condition for all incentive-compatible and individually rational diffusion auctions.
- propose a class of natural monotonic allocation policies with optimal payment policy that maximizes the seller's revenue.

The sufficient and necessary condition

Theorem:

- A diffusion auction (π, x) is incentive-compatible and individually rational if and only if for all type profile $\mathbf{t}$ and all i, $P1 - P5$ are satisfied, where
  - $P1$: π is value-monotonic,
  - $P2$: $\tilde{x}_i$ and $\bar{x}_i$ are bid-independent,
  - $P3$: $\tilde{x}_i(r_i) - \bar{x}_i(r_i) = v_i^*(r_i)$,
  - $P4$: $\tilde{x}_i$ and $\bar{x}_i$ are diffusion-monotonic,
  - $P5$: $\bar{x}_i(\emptyset) \leq 0$.

Open Questions

- More general settings
  - characterize truthful diffusion mechanisms, revenue monotonicity is the key?
- When there is a diffusion cost
  - how to guarantee the seller will not lose?
- Privacy concern and the seller's strategies
  - the seller discovery the whole network and she may cheat as well!
- False-name manipulations
  - a node may create multiple ids as her neighbours to gain more payment?
- many more...

Outline

Diffusion Mechanism Design for Task Allocation

- Wen Zhang, Yao Zhang, Dengji Zhao: *Collaborative Data Acquisition*. AAMAS'20.
- Yao Zhang, Xiuzhen Zhang, Dengji Zhao: *Sybil-proof Answer Querying Mechanism*. IJCAI'20.

Diffusion Mechanism Design for Task Allocation

Resource allocation vs task allocation:

1. task requires more participants' contribution (collaboration)
2. but participants' contribution may conflict with each other (competition)

Crowdsourcing Data Acquisition [Zhang et al. AAMAS'20]

- a requester is collecting data from the crowd
- more participants gives richer dataset
- participants' contribution depends on the quality of their provided data
- if two participants offer the same data, how to calculate their contribution?

Crowdsourcing Data Acquisition [Zhang et al. AAMAS'20]

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Shapley Value?

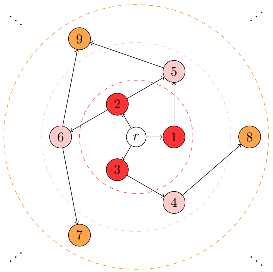
Shapley Value

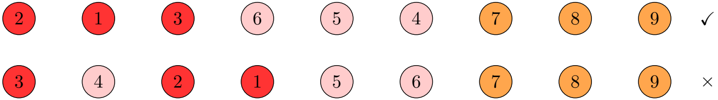
The problem of the Shapley value:

- two participants offer the same data will share the same Shapley value
  - the Shapley value is doubled if one of them didn't participate

Solution: Layered Shapley Value

- participants are layered
- the Shapley value is calculated for each lower layer first
- the calculation for higher layer assumes that lower layers' participants are always in the coalition





Solution: Layered Shapley Value

$$\hat{\phi}_i = \sum_{S \subseteq L_{l_i} - \{i\}} \frac{|S|!(|L_{l_i}| - |S| - 1)!}{|L_{l_i}|!} \cdot \left(v\left(D'_{L^*_{l_i-1} \cup S \cup \{i\}}\right) - v\left(D'_{L^*_{l_i-1} \cup S}\right) \right)$$

Properties:

- participants are incentivized to invite more participants (new participants do not compete with them)
- the requester does not need to pay for redundant data

Sybil-proof Answer Querying [Zhang et al. IJCAI'20]

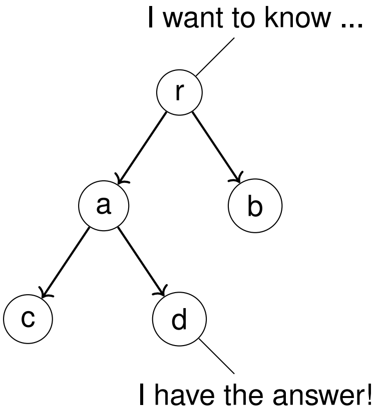


Figure: Query via Network

- Online networks has offered many opportunities for people to collaborate remotely in real time, e.g. P2P file sharing and Q&A platforms.
- Utilizing the social connections, we can enhance the power of answer querying via networks, e.g. DARPA Red Balloon Challenge.

Sybil-proof Answer Querying

- **Fact**: An SIR path mechanism cannot be both SP and CP.
- What if relax SIR to IR?
- **Theorem**: A path mechanism is IR, SP and CP if and only if it is a two-headed mechanism.

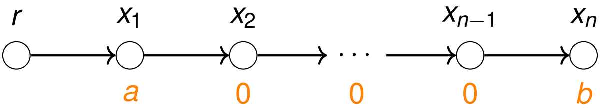


Figure: The rewards distributed by a two-headed mechanism

Sybil-proof Answer Querying

- What if relax CP to λ-CP?
- **New Idea**: Double Geometric Mechanism

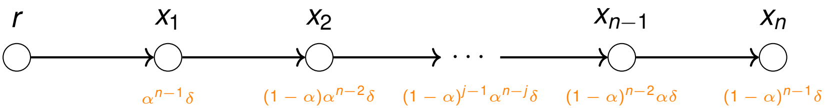


Figure: The rewards distributed by a double geometric mechanism

- **Characterization**: Under mild condition, the properties of IC, SIR, BC, SP, 2-CP and ρ-SS determines a DGM.

Outline

1. Mechanism Design Review
   - The History
   - Second Price Auction (VCG)
2. Diffusion Mechanism Design
   - Resource Allocation
   - Task Allocation
   - Information Propagation

Mechanism Design for Information Propagation

- Haomin Shi, Yao Zhang, Zilin Si, Letong Wang, Dengji Zhao: *Maximal Information Propagation with Budgets*. ECAI'20.

Maximal Information Propagation with Budgets

- The sponsor s wants to propagate some information to the social network modelled as a directed acyclic graph $G = (N, E)$.
- The sponsor holds a fixed budget B, which is prepared as agents' rewards.

Challenge

How to find a reward scheme that is propagation incentive compatible and budget balanced?

Maximal Information Propagation with Budgets

- Budget Distribution Scheme

Budget Distribution Scheme

INPUT: the graph G and the budget $\mathcal{B}$.

1. Using breadth first search to compute the layer sets $L_1, L_2, \ldots, L_{l_{\max}}$ and $L_{l_{\max+1}}$.
2. For each $i \in L_1$, set $b'_i = \mathcal{B}/|L_1|$.
3. For each l in $\{1, \ldots, l_{\max}\}$
   (a) For each $i \in L_l$ compute A_i according to its propagation.
   (b) Let $B_l = (1-\beta)\sum_{i\in L_l} b'_i + \beta \sum_{i\in L_l} A_i b'_i$ and $B'_{l+1} = \sum_{i\in L_l} b'_i - B_l$.
   (c) Distribute B_l to agents in L_l, i.e., for agent i in L_l, she gets r_i as reward.
   (d) Distribute B'_{l+1} to agents in L_{l+1}, i.e., for agent j in L_{l+1}, she gets b'_j as current reward.

OUTPUT: the reward vector $\mathbf{r}$.

- Parameterize the distribution between 2 layers;
- Split the origin amount in the upper layer into 2 parts;
- The budget distribution scheme is IR and WBB.

Maximal Information Propagation with Budgets

- Budget Distribution Scheme Instance

Distribution Algorithm between Two Adjacent Layers

INPUT: the graph G and b'_i for each $i \in L_l$.

1. For each agent $i \in L_l$, set $r_i = i.V_b + i.V_h$, initialize $i.V_b = b'_i$ and $i.V_h = 0$.
2. For each agent $j \in L_{l+1}$, initialize $b'_j = 0$.
3. For each agent $j \in L_{l+1}$
   (a) Let P be the set of agents in L_l who propagate information to j.
   (b) For each agent $i' \in P$:
   - For each agent $i \in L_l \setminus \{i'\}$, set

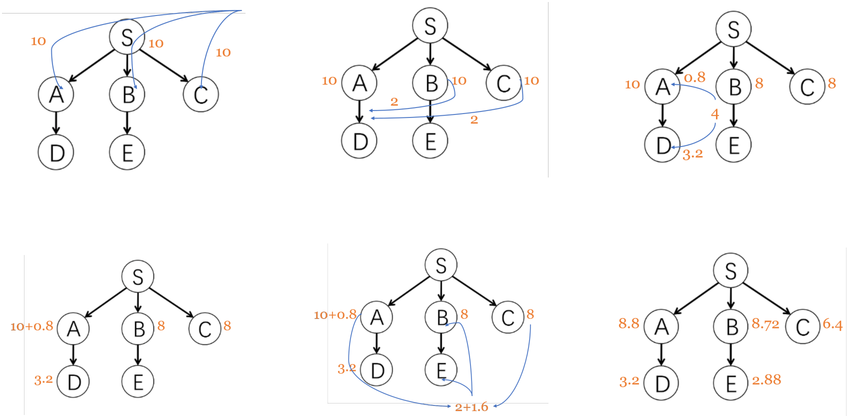
$$i'.V_h \leftarrow i'.V_h + \alpha\beta \cdot i.V_b$$
$$b'_j \leftarrow b'_j + \alpha(1-\beta) \cdot i.V_b$$
$$i.V_b \leftarrow i.V_b - \alpha \cdot i.V_b$$

OUTPUT: r_i for each agent $i \in L_l$ and b'_j for each agent $j \in L_{l+1}$.

- Incentivize propagation based on competition in the same level;
- Agents will get extra reward for their invitation from their competitors;
- This instance is IR, BB, and PIC.

Maximal Information Propagation Example

- Example: Incentivies from Peer Pressure



What we have covered

Mechanism Design Powered by Social Interactions

- Diffusion Mechanism for Resource Allocation (competitive environment)
  - for selling single and multiple items
- Diffusion Mechanism for Task Allocation (both competitive and collaborative)
  - crowdsourcing, sybil-proof, execution uncertainty
- Diffusion Mechanism for Information Propagation
  - information propagation with budgets

http://dengji-zhao.net/ijcai19.html