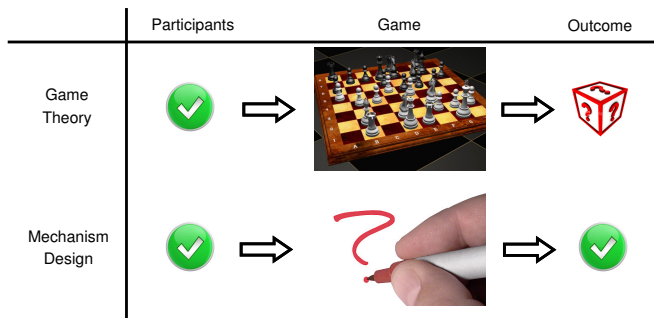


CS243: Introduction to Algorithmic Game Theory

Week 2.2, Mechanism Design (Dengji ZHAO)

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Recap: Game Theory



Recap: (Simultaneous Move) Game Playing

- A set of n players
- Each player i has a set of strategies S_i
- Let $s = (s_1, \dots, s_n)$ be the vector of strategies selected by the n players. Also let $s = (s_i, s_{-i})$.
- Let $S = \times_i S_i$ be the strategy vector space of all players.
- Each $s \in S$ determines the outcome for each player, denote $u_i(s)$ the utility of player i under s .

Recap: (Simultaneous Move) Game Playing

Definition

A strategy vector $s \in S$ is a **dominant strategy**, if for each player i , and each alternate strategy vector $s' \in S$, we have that $u_i(s_i, s'_{-i}) \geq u_i(s'_i, s'_{-i})$.

Definition

A strategy vector $s \in S$ is said to be a (pure strategy) **Nash equilibrium** if for all players i and each alternate strategy $s'_i \in S_i$, we have that $u_i(s_i, s_{-i}) \geq u_i(s'_i, s_{-i})$.

Definition

We say that a change from strategy s_i to s'_i is an **improving response** for player i if $u_i(s'_i, s_{-i}) > u_i(s_i, s_{-i})$ and **best response** if s'_i maximizes the players' utility $\max_{s'_i \in S_i} u_i(s'_i, s_{-i})$.

Recap: Auction Design

- **Second Price Auction** (Vickrey Auction)
 - Each buyer reports her valuation to the seller
 - The seller sells the item to the buyer with the highest valuation report
 - The seller charges the winner the second highest valuation report

Definition

An auction is **truthful** if reporting valuation truthfully is a **dominant strategy** for all participants.

Questions

Questions

- Is there any weakness of truthfulness?
- How to extend second price auction to multiple items settings?

The General Setting of Mechanism/Auction Design

- A set of n participants/players, denoted by N .
- A mechanism needs to choose some alternative from A (allocation space), and to decide a payment for each player.
- Each player $i \in N$ has a **private** valuation function $v_i : A \rightarrow \mathbb{R}$, let V_i denote all possible valuation functions for i .
- Let $v = (v_1, \dots, v_n)$, $v_{-i} = (v_1, \dots, v_{i-1}, v_{i+1}, \dots, v_n)$.
- Let $V = V_1 \times \dots \times V_n$, $V_{-i} = V_1 \times \dots \times V_{i-1} \times V_{i+1} \times \dots \times V_n$.

A Definition of a Mechanism (with Money)

Definition

A (direct revelation) **mechanism** is a **social choice function** $f : V_1 \times \cdots \times V_n \rightarrow A$ and a vector of **payment functions** p_1, \dots, p_n , where $p_i : V_1 \times \cdots \times V_n \rightarrow \mathbb{R}$ is the amount that player i pays.

- **direct revelation**: *the mechanism requires each player to report her valuation function to the mechanism.*

Definition

Given a mechanism (f, p_1, \dots, p_n) , and players' valuation report profile $v' = (v'_1, \dots, v'_i, v'_n)$, player i 's **utility** is defined by $v_i(f(v')) - p_i(v')$, where v_i is i 's true valuation function.

Properties of a Mechanism: Truthfulness

Definition

A mechanism (f, p_1, \dots, p_n) is called **truthful** (*incentive compatible*) if for every player i , every $v_1 \in V_1, \dots, v_n \in V_n$ and every $v'_i \in V_i$, we have

$$v_i(a) - p_i(v_i, v_{-i}) \geq v_i(a') - p_i(v'_i, v_{-i})$$

where $a = f(v_i, v_{-i})$ and $a' = f(v'_i, v_{-i})$.

- $v_i(a) - p_i(v_i, v_{-i})$ is i 's **utility** to report v_i
- $v_i(a') - p_i(v'_i, v_{-i})$ is i 's utility to report v'_i

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- $v_i(a) - p_i(v_i, v_{-i})$ is i 's **utility** to report v_i
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Question

A mechanism is *truthful* means that reporting valuation function truthfully is a *dominant strategy* for all players?

How to verify a mechanism is truthful or not?

Theorem

A mechanism is truthful *if and only if* it satisfies the following conditions for every i and every v_{-i} :

- 1 *The payment p_i does not depend on v_i , but only on the alternative chosen $f(v_i, v_{-i})$. That is, for every v_{-i} , there exist prices $p_a \in \mathbb{R}$, for every $a \in A$, such that for all v_i with $f(v_i, v_{-i}) = a$ we have that $p(v_i, v_{-i}) = p_a$.*
- 2 *The mechanism optimizes for each player. That is, for every v_i , we have that $f(v_i, v_{-i}) \in \arg \max_a (v_i(a) - p_a)$, where the quantification is over all alternatives in the range of $f(\cdot, v_{-i})$.*

Questions

Questions

- How to prove first price auction is NOT truthful?

Questions

Questions

- How to prove first price auction is NOT truthful?
- How to prove fixed price auction is truthful (with randomized tie-breaking)?

Properties of a Mechanism: Efficiency

Definition

Given an alternative $a \in A$, the **social welfare** of choosing a is $\sum_{i \in N} v_i(a)$.

Definition (Efficiency)

We say a social choice function f is **efficient** if it maximises social welfare for all valuation reports. That is, for all $v \in V$,

$$f \in \arg \max_{f' \in F} \sum_{i \in N} v_i(f'(v))$$

where F is the set of all **feasible** social choice functions.

Questions

- Is the second price auction efficient?
- Is the first price auction efficient?
- Is fixed price auction efficient?

Questions

- Is the second price auction efficient? Yes!
- Is the first price auction efficient? Yes!
- Is fixed price auction efficient? No!

Properties of a Mechanism: Individual Rationality

Definition

Given a mechanism (f, p_1, \dots, p_n) , a valuation report profile v' , a player i 's **utility** is **quasi-linear** and is defined by

$$u_i(f, p_1, \dots, p_n, v', v_i) = v_i(f(v')) - p_i(v')$$

Definition

We say a mechanism (f, p_1, \dots, p_n) is **individually rational** if for every player i , every $v \in V$, we have $u_i(f, p_1, \dots, p_n, v, v_i) \geq 0$.

Properties of a Mechanism: Individual Rationality

Definition

Given a mechanism (f, p_1, \dots, p_n) , a valuation report profile v' , a player i 's **utility** is **quasi-linear** and is defined by

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Definition

We say a mechanism (f, p_1, \dots, p_n) is **individually rational** if for every player i , every $v \in V$, we have $u_i(f, p_1, \dots, p_n, v, v_i) \geq 0$.

- *That is, players are **not forced to participate** in the mechanism.*

Questions

- Is the second price auction individually rational?
- Is the first price auction individually rational?
- Is fixed price auction individually rational?

Questions

- Is the second price auction individually rational? Yes!
- Is the first price auction individually rational? Yes!
- Is fixed price auction individually rational? Yes!

Vickrey-Clarke-Groves Mechanism

- The setting:
 - A set of m items to be allocated (denoted by M)
 - A set of n players (denoted by N)
 - Each player i has a valuation function $v_i : 2^M \rightarrow \mathbb{R}$
- VCG:
 - Choose an efficient allocation
 - Charge each player the social welfare loss of the others due to her participation

Advanced Reading

- Introduction to Mechanism Design [AGT Chapter 9]
- Vickrey-Clarke-Groves mechanisms [AGT Chapter 9.3]