CS243: Introduction to Algorithmic Game Theory

Week 2.1, Dominate Strategy and Truthfulness (Dengji ZHAO)

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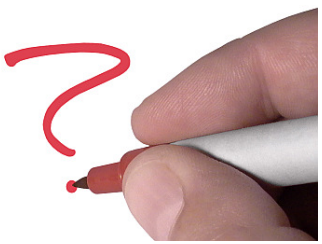
Recap: Game Theory

	Participants	Game	Outcome
Game Theory			
Mechanism Design			

Recap: (Simultaneous Move) Game Playing

- A set of n players
- Each player i has a set of strategies S_i
- Let $s = (s_1, \cdots, s_n)$ be the vector of strategies selected by the n players. Also let $s = (s_i, s_{-i})$.
- Let $S = \prod_i S_i$ be the strategy vector space of all players.
- Each $s \in S$ determines the outcome for each player, denote $u_i(s)$ the utility of player i under s.

Recap: (Simultaneous Move) Game Playing

Definition

A strategy vector $s \in S$ is a dominant strategy equilibrium, if for each player i, and each alternate strategy vector $s' \in S$, we have that $u_i(s_i, s'_{-i}) \geq u_i(s'_i, s'_{-i})$

Definition

A strategy vector $s \in S$ is said to be a (pure strategy) Nash equilibrium if for all players i and each alternate strategy $s'_i \in S_i$, we have that $u_i(s_i, s_{-i}) \geq u_i(s'_i, s_{-i})$

Recap: Games

P1 \ P2	Confess	Silent
Confess	4, 4	5, 1
Silent	1, 5	2, 2

Prisoners' Dilemma

Girl \ Boy	B	S
B	6, 5	1, 1
S	2, 2	5, 6

Battle of the Sexes

1 \ 2	H	T
H	-1, 1	1, -1
T	1, -1	-1, 1

Matching Pennies

How to compute strategies?

Learning in Games: Best Response

Best Response

Definition

We say that a change from strategy s_i to s_i' is an improving response for player i if $u_i(s_i', s_{-i}) > u_i(s)$ and best response if s_i' maximizes the players' utility $\max_{s_i' \in S_i} u_i(s_i', s_{-i})$.

Best Response

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Game Design: Mechanism Design

- Auctions (Second Price Auction)

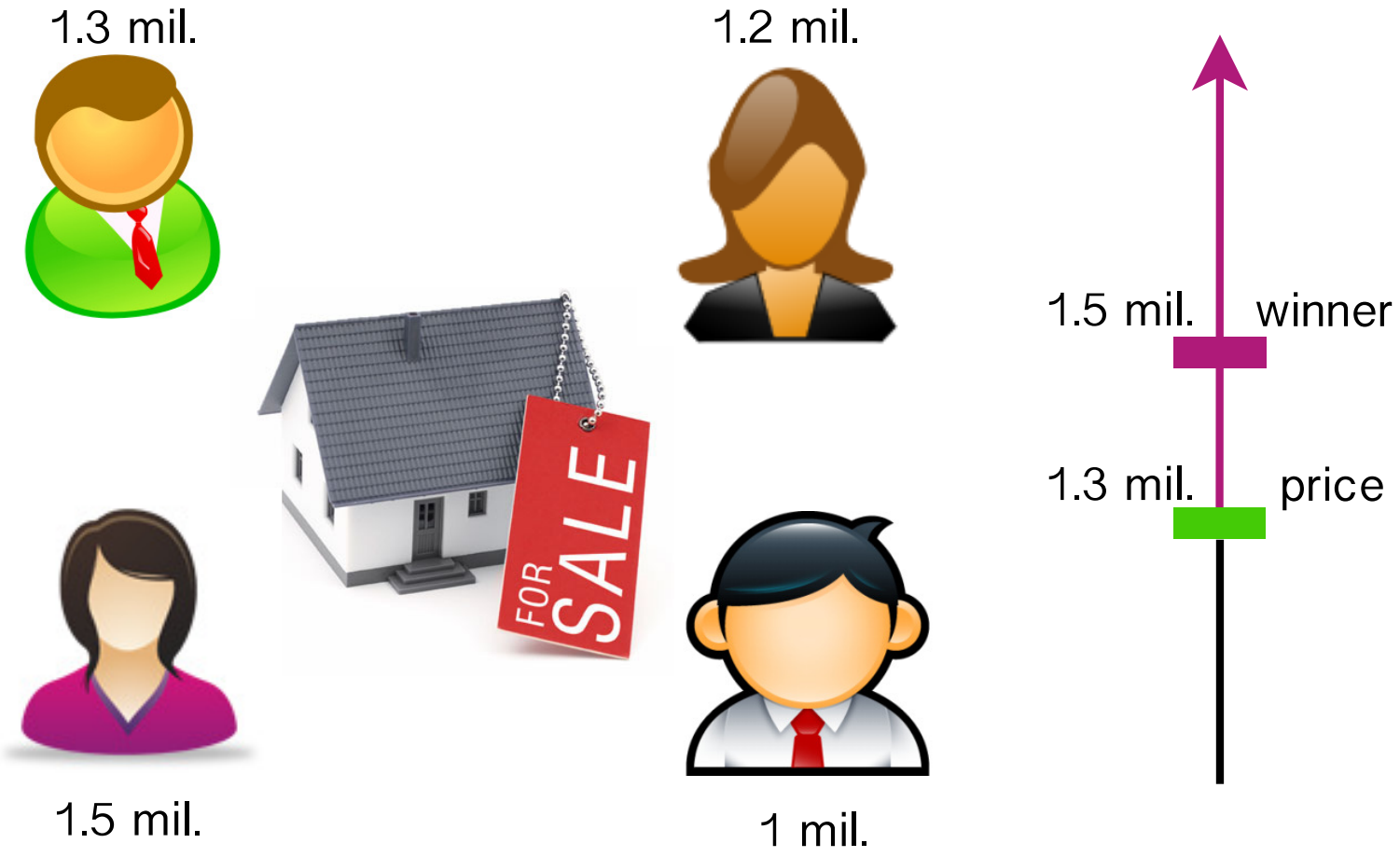
The Setting

- A seller sells an item, e.g. a house.
- A set of n buyers are willing to buy the item, each buyer i has a (private) valuation v_i on the item.

Second Price Auction (Vickrey Auction)

- Each buyer reports her valuation to the seller
- The seller sells the item to the buyer with the highest valuation report
- The seller charges the winner the second highest valuation report

Second Price Auction (Vickrey Auction)



Strategies of the Buyers

- Strategy/Action space:

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- Strategy/Action space:
- What is the best strategy for a buyer?

Dominant Strategy in Auction Design: Truthfulness

Definition

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  - Receive truthful valuation information for other decision making, e.g. maximising social welfare

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Question

Is there any weakness of truthfulness?

Challenges

Challenge

Is first price auction truthful?

Challenges

Challenge

Is first price auction truthful?

Question

Is fixed price auction truthful?

- A fixed price is given in advance/public-known.
- All buyers whose reports above the fixed prices will win and pay the fixed price.
- If the number of buyers above the price is more than the number of items to sell, use random tie-breaking.

Advanced Reading

Challenge

How to extend second price auction for single item to multiple items settings? Vickrey-Clarke-Groves (VCG)

- Introduction to Mechanism Design [AGT Chapter 9]