CS243: Introduction to Algorithmic Game Theory

Week 1.2, Basic Concepts (Dengji ZHAO)

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Outline

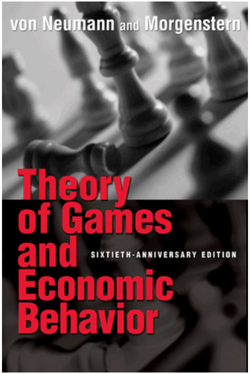
1. Recap
2. Basic Concepts

Announcements

1. Piazza is online!

Recap: What is Game Theory

- **Game theory** is the study of mathematical models of conflict and cooperation between intelligent rational decision-makers [von Neumann and Morgenstern 1944].



- Extensive form: Go, poker
- Normal form: rock-paper-scissors
- Cooperative game: coordination games

Recap: What is Game Theory

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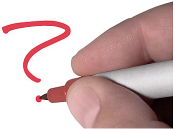
	Participants	Game	Outcome
Game Theory	✓		
Mechanism Design	✓		✓

Mechanism Design (Reverse Game Theory)

Mechanism Design is to answer...

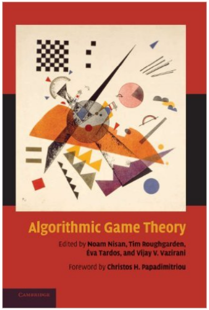
Question

How to design a mechanism/game, toward desired objectives, in strategic settings?

- **Roger B. Myerson** (born March 29, 1951, University of Chicago, US)
  - Nobel Prize for economics (2007), for "having laid the foundations of mechanism design theory."
  - *Eleven game-theorists have won the economics Nobel Prize*.

When Game Theory Meets CS?

- **Algorithmic Game Theory** is an area in the intersection of game theory and algorithm design, whose objective is to design algorithms in strategic environments [Nisan et al. 2007].



- *Computing in Games*: algorithms for computing equilibria
- *Algorithmic Mechanism Design*: design games that have both good game-theoretical and algorithmic properties
- ...

When Game Theory Meets CS?

- **Algorithmic Game Theory** is an area in the intersection of game theory and algorithm design, whose objective is to design algorithms in strategic environments [Nisan et al. 2007].

It is multidisciplinary:

- Artificial Intelligence → Multi-agent Systems → Algorithmic Game Theory
- Economics
- Theoretical Computer Science

Algorithmic Game Theory in Artificial Intelligence

- Algorithmic Game Theory research in AI (multi-agent systems):
  - Game Playing: computation challenge, AlphaGo, poker
  - Social Choice: preferences aggregation, voting, prediction
  - Mechanism Design: the allocation of scarce resources (security games), Ad auctions, online auctions, false-name-proof mechanisms (**Makoto Yokoo**)
- IJCAI Computers and Thought Award: 5 out of the 12 winners (1999-2017) had worked on AGT, **Nick Jennings** (1999), Tuomas Sandholm (2003), Peter Stone (2007), Vice Conitzer (2011), Ariel Procaccia (2015).

Outline

1. Recap
2. Basic Concepts
    - Classical Games
    - Solution Concepts

Prisoners’ Dilemma

- Two players: P1 and P2
- Strategies: Confess, Silent
- Outcomes: number of years in prison

P2 P1	Confess	Silent
Confess	4 4	5 1
Silent	1 5	2 2

Battle of the Sexes

- Two players: Girl, Boy
- Strategies: Baseball (B), Softball (S)
- Outcomes: payoffs/benefits/utilities

	Boy	
Girl	B	S
B	6 5	1 1
S	2 2	5 6

Simultaneous Move Game

- A set of n players
- Each player i has a set of strategies S_i
- Let $s = (s_1, \cdots, s_n)$ be the vector of strategies selected by the n players. Also let $s = (s_i, s_{-i})$.
- Let $S = \prod_i S_i$ be the strategy vector space of all players.
- Each $s \in S$ determines the outcome for each player, denote $u_i(s)$ the utility of player i under s.

Dominant Strategy

Definition

A strategy $s_i \in S_i$ is a dominant strategy for player i, if for all $s' \in S$, we have that $u_i(s_i, s'_{-i}) \geq u_i(s'_i, s'_{-i})$

Definition

A strategy vector $s \in S$ is a dominant strategy equilibrium, if for each player i, and each alternate strategy vector $s' \in S$, we have that $u_i(s_i, s'_{-i}) \geq u_i(s'_i, s'_{-i})$

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Any dominant strategy equilibrium in Prisoners' Dilemma?

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A strategy vector $s \in S$ is said to be a Nash equilibrium if for all players i and each alternate strategy $s'_i \in S_i$, we have that

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Any Nash equilibrium in Battle of the Sexes?

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Another Game: Matching Pennies

- Two players: 1, 2
- Strategies: Head (H), Tail (T)
- Outcomes: the row player (1) wins if the two pennies match, while the column player wins if they do not match

2 1	H	T
H	-1 1	1 -1
T	1 -1	-1 1

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- Two players: 1, 2
- Strategies: Head (H), Tail (T)
- Outcomes: the row player (1) wins if the two pennies match, while the column player wins if they do not match
- Any dominant strategy or Nash equilibrium?

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Mixed Strategies

Definition

Each player i picks a probability distribution p_i over his set of possible strategies S_i, such a choice is called a mixed strategy.

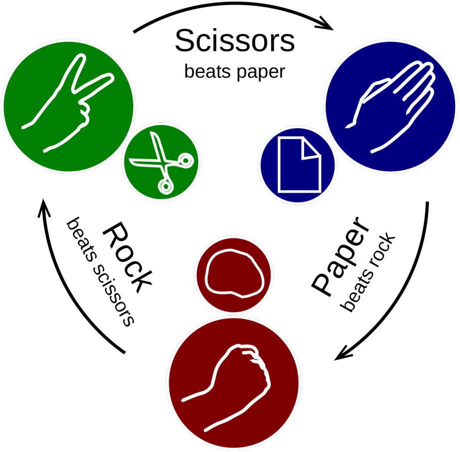
- Given a player i's probability distribution choice p_i over S_i, let $p_i(s_i)$ be the probability to choose strategy s_i, we have $\sum_{s_i \in S_i} p_i(s_i) = 1$.
- Assume that players are risk-neutral; that is, they act to maximize the expected payoff.

Mixed Strategy Nash Equilibrium

- Two players: 1, 2
- Strategies: Head (H), Tail (T)
- Outcomes: the row player (1) wins if the two pennies match, while the column player wins if they do not match
- If player 1 uses mixed strategy $p_1(H) = p_1(T) = 0.5$, what is the best strategy for player 2?

2 1	H	T
H	-1 1	1 -1
T	1 -1	-1 1

Mixed Strategy Nash Equilibrium



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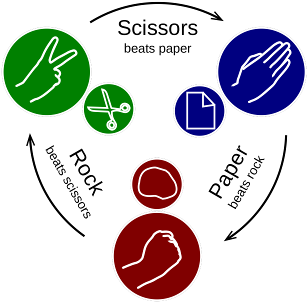

Quiz

If one player can only choose Rock and Paper, what is the best strategy for the other player?

Advanced Reading

- Games with no Nash equilibria [AGT Chapter 1.3.5]
- Correlated Equilibrium [AGT Chapter 1.3.6]