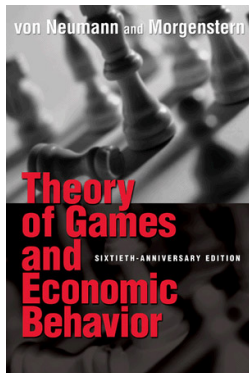# CS243: Introduction to Algorithmic Game Theory

Week 1.1, Introduction (Dengji ZHAO)

SIST, ShanghaiTech University, China

What is Game Theory

- **Game theory** is the study of mathematical models of conflict and cooperation between intelligent rational decision-makers [von Neumann and Morgenstern 1944].

- Extensive form: Go, poker
- Normal form: rock-paper-scissors
- Cooperative game: coordination games

What is Game Theory

- **Game theory** is the study of mathematical models of conflict and cooperation between intelligent rational decision-makers [von Neumann and Morgenstern 1944].

	Participants	Game	Outcome
Game Theory			
Mechanism Design			
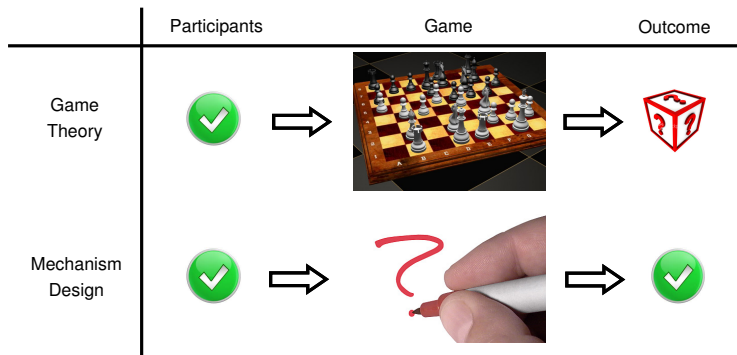
Mechanism Design (Reverse Game Theory)

Mechanism Design is to answer...

Question

How to design a mechanism/game, toward desired objectives, in strategic settings?

- **Roger B. Myerson** (born March 29, 1951, University of Chicago, US)
  - Nobel Prize for economics (2007), for "having laid the foundations of mechanism design theory."
  - *Eleven game-theorists have won the economics Nobel Prize.*

When Game Theory Meets CS?

- **Algorithmic Game Theory** is an area in the intersection of game theory and algorithm design, whose objective is to design algorithms in strategic environments [Nisan et al. 2007].
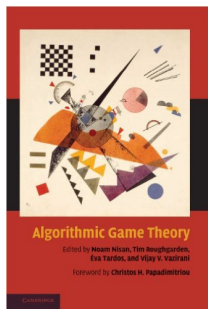
- *Computing in Games*: algorithms for computing equilibria
- *Algorithmic Mechanism Design*: design games that have both good game-theoretical and algorithmic properties
- ...

When Game Theory Meets CS?

- **Algorithmic Game Theory** is an area in the intersection of game theory and algorithm design, whose objective is to design algorithms in strategic environments [Nisan et al. 2007].

It is multidisciplinary:

- Artificial Intelligence $\rightarrow$ Multi-agent Systems $\rightarrow$ Algorithmic Game Theory
- Economics
- Theoretical Computer Science

Algorithmic Game Theory in Artificial Intelligence

- Algorithmic Game Theory research in AI (multi-agent systems):
  - Game Playing: computation challenge, AlphaGo, poker
  - Social Choice: preferences aggregation, voting, prediction
  - Mechanism Design: the allocation of scarce resources (security games), Ad auctions, online auctions, false-name-proof mechanisms (**Makoto Yokoo**)

- IJCAI Computers and Thought Award: 5 out of the 12 winners (1999-2017) had worked on AGT, **Nick Jennings** (1999), Tuomas Sandholm (2003), Peter Stone (2007), Vice Conitzer (2011), Ariel Procaccia (2015).
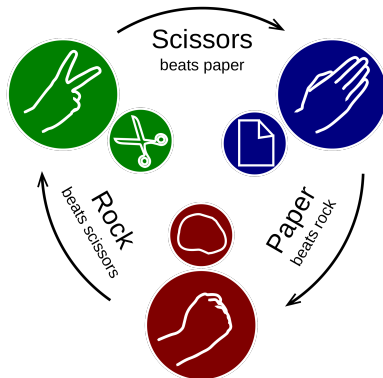
Outline

Game Play

Let's playing games...

Rock Paper Scissors

Prisoners' Dilemma

- Two players: P1 and P2
- Strategies: Confess, Silent
- Outcomes: number of years in prison

		P2	
P1		Confess	Silent
Confess		4 / 4	5 / 1
Silent		1 / 5	2 / 2

Battle of the Sexes

- Two players: Girl, Boy
- Strategies: Baseball (B), Softball (S)
- Outcomes: payoffs/benefits/utilities

	Boy	
Girl	B	S
B	6 5	1 1
S	2 2	5 6

Chicken Game

	Swerve	Straight
Swerve	Tie, Tie	Lose, Win
Straight	Win, Lose	Crash, Crash

- Two players: P1, P2
- Strategies: Swerve, Straight
- Outcomes: utilities

	Swerve	Straight
Swerve	0, 0	-1, +1
Straight	+1, -1	-1000, -1000

Ultimatum Game

- Two players: P1, P2
- Strategies:
  - P1: propose to divide $100 between P1 and P2
  - P2: accept or reject
- Outcomes: utilities
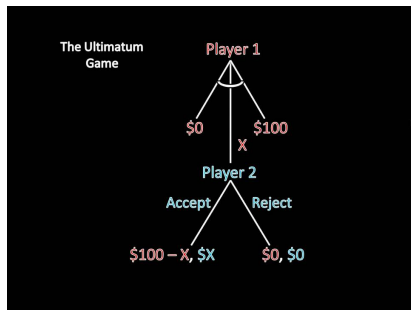
College/Master/PhD Entrance Exam

The setting:

- The are two colleges: C_1, C_2 and three students S_1, S_2, S_3.
- The colleges' preferences are:
  - C_1: $S_1 \succ S_2 \succ S_3$
  - C_2: $S_1 \succ S_2 \succ S_3$
- The students' preferences are:
  - S_1: $C_1 \succ C_2$
  - S_2: $C_1 \succ C_2$
  - S_3: $C_2 \succ C_1$

The process (each college can only accept one student):

1. Stage 1: each student chooses one college to apply and each college chooses her most preferred student to accept.

2. Stage 2: unaccepted students from Stage 1 can choose their second college to apply and the colleges who have quota left choose their most preferred student to accept.

Outline

A Game Design Example

Let's design a game...

Design Goal

How can a house-seller sell her house with the "highest" profit?

A Game Design Example

> **Design Goal**
>
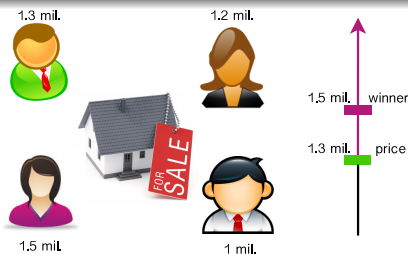> How can a house-seller sell her house with the "highest" profit?

- **Challenge**: the seller doesn't know how much the buyers are willing to pay (their valuations).

A Game Design Example

Design Goal

How can a house-seller sell her house with the "highest" profit?

Solution: Second Price Auction (Vickrey Auction/VCG)

- Input: each buyer reports a price/bid to the seller
- Output: the seller decides
  - *allocation*: the agent with the highest report price wins.
  - *payment*: the winner pays the second highest price reported.

A Game Design Example

Design Goal
How can a house-seller sell her house with the "highest" profit?
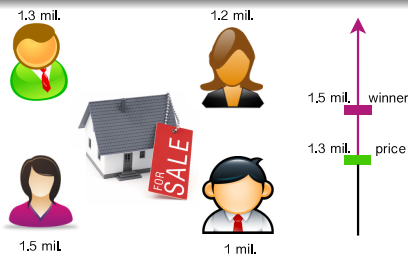
Solution: Second Price Auction (Vickrey Auction/VCG)

Properties:

- Efficient: maximising social welfare
- Truthful: buyers will report their highest willing payments

Generalized Second Price (GSP) Auction

Generalized Second Price (GSP) Auction

Advertiser	Bid	Price	Order	CTR
A_1	6	4	1st	1
A_2	4	2	2nd	0.9
A_3	2	1	3rd	0.5
A_4	1			

Car Plates Allocation